

Answers & Solutions

# JEE Advanced Paper-2

# MOMENTUM

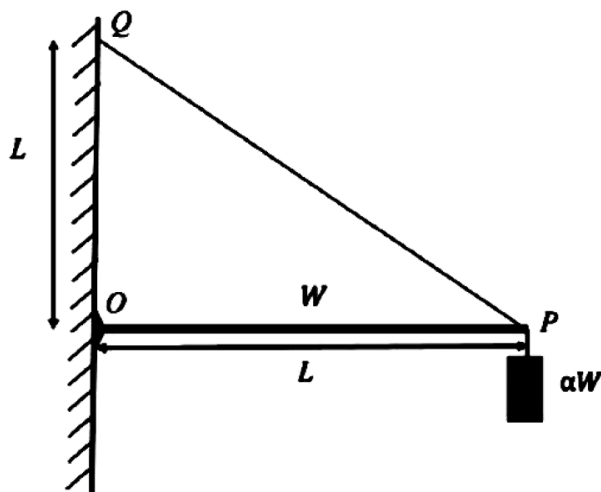
## PART-1: PHYSICS

### SECTION-1(Maximum Marks: 24)

#### One or More Type

- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s)
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:  
Full Marks : +4 If only (all) the correct option(s) is(are) chosen;  
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;  
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;  
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;  
Zero Marks : 0 If unanswered;  
Negative Marks : -2 In all other cases.

1. One end of a horizontal uniform beam of weight  $W$  and length  $L$  is hinged on a vertical wall at point  $O$  and its other end is supported by a light inextensible rope. The other end of the rope is fixed at point  $Q$ , at a height  $L$  above the hinge at point  $O$ . A block of weight  $\alpha W$  is attached at the point  $P$  of the beam, as shown in the figure (not to scale). The rope can sustain a maximum tension of  $(2\sqrt{2})W$ . Which of the following statement(s) is(are) correct?



- A) The vertical component of reaction force at  $O$  does **not** depend on  $\alpha$
- B) The horizontal component of reaction force at  $O$  is equal to  $W$  for  $\alpha = 0.5$
- C) The tension in the rope is  $2W$  for  $\alpha = 0.5$
- D) The rope breaks if  $\alpha > 1.5$

**Ans. ABD**

# MOMENTUM



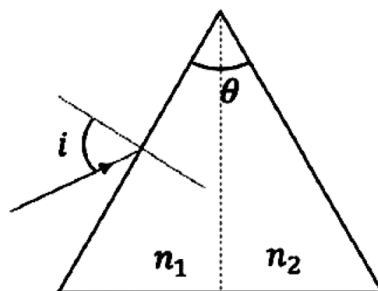
# MOMENTUM

$$\text{For } f_s = \frac{f_0}{2} \Rightarrow \lambda_{\text{app}} = \frac{V}{10f_0} \neq \lambda_{\text{res}}$$

$$\text{For } f_s = \frac{f_0}{2} \Rightarrow \lambda_{\text{app}} = \frac{2V}{5f_s} \neq \lambda_{\text{res}}$$

$$\text{For } u = \frac{V}{2} \text{ and } f_s = \frac{3f_0}{2} \Rightarrow \lambda_{\text{app}} = \frac{V}{2f_s} = \frac{V}{3f_0} = \lambda_{\text{res}}$$

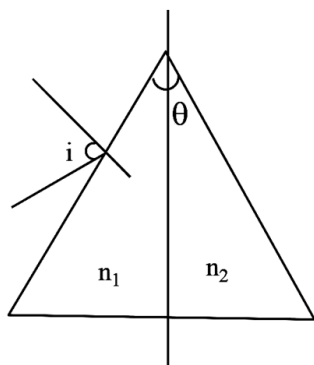
3. For a prism of prism angle  $\theta = 60^\circ$ , the refractive indices of the left half and the right half are, respectively,  $n_1$  and  $n_2$  ( $n_2 \geq n_1$ ) as shown in the figure. The angle of incidence  $i$  is chosen such that the incident light rays will have minimum deviation if  $n_1 = n_2 = n = 1.5$ . For the case of unequal refractive indices,  $n_1 = n$  and  $n_2 = n + \Delta n$  (where  $\Delta n \ll n$ ), the angle of emergence  $e = i + \Delta e$ . Which of the following statement(s) is(are) correct ?



- A) The value of  $\Delta e$  (in radians) is greater than that of  $\Delta n$
- B)  $\Delta e$  is proportional to  $\Delta n$
- C)  $\Delta e$  lies between 2.0 and 3.0 milliradians, if  $\Delta n = 2.8 \times 10^{-3}$
- D)  $\Delta e$  lies between 1.0 and 1.6 milliradians, if  $\Delta n = 2.8 \times 10^{-3}$

**Ans. BC**

**Sol.**  $i = \frac{A+D}{2}$



$$\frac{3}{2} = \frac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}} \quad \sin \left( \frac{A+D}{2} \right) = \frac{3}{4}$$

# MOMENTUM

$$i_1 + i_2 = A + D \text{ and } r_1 - r_2 = \frac{A}{2}$$

$$\text{as } n = \frac{\sin i}{\sin r} \Rightarrow \sin e = \frac{n + \Delta n}{2} \Rightarrow \sin(i + \Delta e) = \frac{n + \Delta n}{2}$$

$$\Rightarrow \sin i (\cos \Delta e) + \cos i \sin(\Delta e) = \frac{n + \Delta n}{2}$$

$$\Rightarrow \frac{n}{2} \times 1 + \frac{\sqrt{3}n}{2} (\Delta e) = \frac{n}{2} + \frac{\Delta n}{2}$$

$$\Delta e = (\Delta n) \frac{2}{\sqrt{3}}$$

$$\text{For } \Delta n = 2.8 \times 10^{-3} \text{ and } n = \frac{3}{2}$$

$$\Delta e = 2.15 \times 10^{-3}$$

4. A physical quantity  $\vec{S}$  is defined as  $\vec{S} = (\vec{E} \times \vec{B}) / \mu_0$ , where  $\vec{E}$  is electric field,  $\vec{B}$  is magnetic field and  $\mu_0$  is the permeability of free space. The dimensions of  $\vec{S}$  are the same as the dimensions of which of the following quantity (ies) ?

A)  $\frac{\text{Energy}}{\text{Charge} \times \text{Current}}$

B)  $\frac{\text{Force}}{\text{Length} \times \text{Time}}$

C)  $\frac{\text{Energy}}{\text{Volume}}$

D)  $\frac{\text{Power}}{\text{Area}}$

**Ans. BD**

**Sol.**  $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} =$

$$[S] = \frac{[E][B]}{[\mu_0]} = \frac{[CB][B]}{[\mu_0]} = \left[ \frac{B^2}{\mu_0} \right] [C]$$

$\frac{B^2}{\mu_0}$  has dimensions of energy density

$$\therefore [S] = \frac{[\text{energy}][\text{speed}]}{[\text{volume}]} = \left( \frac{ML^2T^{-2}}{(L^3)(LT^{-1})} \right) = MT^{-3}$$

5. A heavy nucleus N, at rest, undergoes fission  $N \rightarrow P + Q$ , where P and Q are two lighter nuclei. Let  $\delta = M_N - M_P - M_Q$ , where  $M_P, M_Q$  and  $M_N$  are the masses of P, Q and N, respectively.  $E_P$  and  $E_Q$  are the kinetic energies of P and Q, respectively. The speeds of P and Q are  $v_P$  and  $v_Q$ , respectively. If c is the speed of light, which of the following statement(s) is(are) correct?

# MOMENTUM

A)  $E_P + E_Q = c^2 \delta$

B)  $E_P = \left( \frac{M_P}{M_P + M_Q} \right) c^2 \delta$

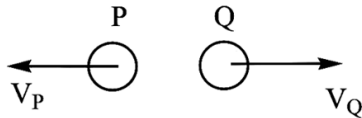
C)  $\frac{v_P}{v_Q} = \frac{M_Q}{M_P}$

D) The magnitude of momentum for P as well as Q is  $C\sqrt{2\mu\delta}$ , where

$$\mu = \frac{M_P M_Q}{(M_P + M_Q)}$$

**Ans. ACD**

**Sol.**  $N \rightarrow P + Q$



$$\delta = M_N - M_P - M_Q \quad Q = \delta c^2$$

$$\therefore E_P + E_Q = \delta c^2$$

Since there are no external forces acting on the system. Momentum has to be conserved.

$$P_P = P_Q = P$$

$$M_P V_P = M_Q V_Q$$

$$\frac{V_P}{V_Q} = \frac{M_Q}{M_P}$$

$$K.E = \frac{P^2}{2M} \quad \therefore \frac{P^2}{2M_P} + \frac{P^2}{2M_Q} = \delta c^2$$

$$P^2 = \frac{2M_P M_Q \cdot \delta c^2}{M_P + M_Q}$$

$$P = \sqrt{2\mu\delta c^2}$$

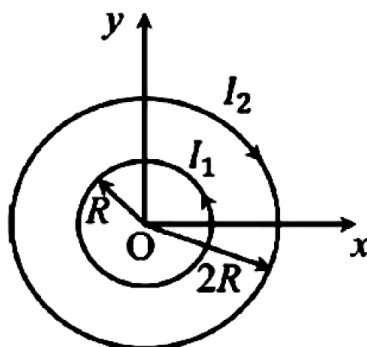
$$P = c\sqrt{2\mu\delta}$$

$$\text{Where } \mu = \frac{M_P M_Q}{M_P + M_Q}$$

6. Two concentric circular loops, one of radius  $R$  and the other of radius  $2R$ , lie in the  $xy$ -plane with the origin as their common centre, as shown in the figure. The smaller

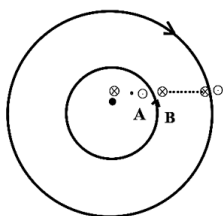
# MOMENTUM

loop carries current  $I_1$  in the anti-clockwise direction and the larger loop carries current  $I_2$  in the clockwise direction, with  $I_2 > 2I_1$ .  $\vec{B}(x, y)$  denotes the magnetic field at a point  $(x, y)$  in the  $xy$ -plane. Which of the following statement(s) is(are) correct?



- A)  $\vec{B}(x, y)$  is perpendicular to the  $xy$ -plane at any point in the plane
- B)  $|\vec{B}(x, y)|$  depends on  $x$  and  $y$  only through the radial distance  $r = \sqrt{x^2 + y^2}$
- C)  $|\vec{B}(x, y)|$  is non-zero at all points for  $r < R$
- D)  $\vec{B}(x, y)$  points normally outward from the  $xy$ -plane for all the points between the two loops

**Ans. AB**



**Sol.**

$$B_{\text{center}} = \frac{\mu_0 i_1}{2R} \odot - \frac{\mu_0 i_2}{4R} \otimes, i_2 > 2i_1 \text{ so } B_{\text{center}} \text{ is } \otimes$$

direction A  $\vec{B}$  changes as we cross any wire

Due to radial symmetry  $B$  depends only on  $r$ .

consider point A just inside inner loop and B just outside it field at A due to inner loop is outward so there must be a null point somewhere between O & A (for  $r < R$ )

Also field between the loops is inward field just outside the outer loop is outward and at far away points it is inwards. So there is another null point outside the outer loop (for  $r > 2R$ )

# MOMENTUM

## SECTION-2(Maximum Marks: 12)

### Paragraph with Numerical

- This section contains THREE (03) question stems.
- There are TWO (02) questions corresponding to each question stem.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;

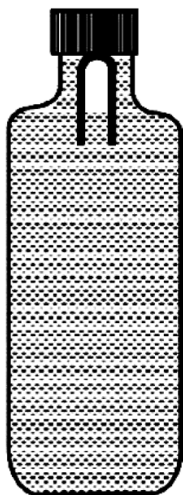
Zero Marks : 0 In all other cases.

### Question Stem for Question Nos. 7 and 8

#### Question Stem

A soft plastic bottle, filled with water of density  $1 \text{ gm/cc}$ , carries an inverted glass test-tube with some air (ideal gas) trapped as shown in the figure. The test-tube has a mass of  $5 \text{ gm}$ , and it is made of a thick glass of density  $2.5 \text{ gm/cc}$ . Initially the bottle is sealed at atmospheric pressure  $p_0 = 10^5 \text{ Pa}$  so that the volume of the trapped air is  $v_0 = 3.3 \text{ cc}$ . When the bottle is squeezed from outside at constant temperature, the pressure inside rises and the volume of the trapped air reduces. It is found that the test tube begins to sink at pressure  $p_0 + \Delta p$  without changing its orientation. At this pressure, the volume of the trapped air is  $v_0 - \Delta v$ .

Let  $\Delta v = X \text{ cc}$  and  $\Delta p = Y \times 10^3 \text{ Pa}$ .



7. The value of X is \_\_\_\_\_.

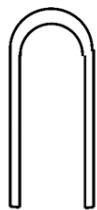
Ans. 0.30

8. The value of Y is \_\_\_\_\_.

Ans. 10.00

# MOMENTUM

## Sol. 7&8



mass test tube = 5g

$$\text{volume} = \frac{5g}{2.5g/cc} = 2cc$$

Buoyant force = 2g

ignoring mass of air trapped, minimum value of volume of air trapped for glass tube to float

$$= \frac{W - B}{S_{\text{water}}} = 3cc$$

$$V_{\text{in}} = 3.3cc \quad V_{\text{fin}} = 3cc \quad \Delta V = 0.3cc = X$$

$$P_{\text{in}} = P_0 \quad P_{\text{fin}} = P_0 + \Delta P$$

isothermal condition, hence

$$P_{\text{in}} V_{\text{in}} = P_{\text{fin}} V_{\text{fin}}$$

$$(3.3)P_0 = 3(P_0 + \Delta P) \quad \Rightarrow \quad P_0 + \Delta P = 1.1P_0$$

$$\Delta P = 0.1P_0 = 10^4 \text{ Pa} \quad \therefore \quad Y = 10$$

## Question Stem for Question Nos. 9 and 10

### Question Stem

A pendulum consists of a bob of mass  $m = 0.1 \text{ kg}$  and a massless inextensible string of length  $L = 1.0 \text{ m}$ . It is suspended from a fixed point at height  $H = 0.9 \text{ m}$  above a frictionless horizontal floor. Initially, the bob of the pendulum is lying on the floor at rest vertically below the point of suspension. A horizontal impulse  $P = 0.2 \text{ kg-m/s}$  is imparted to the bob at some instant. After the bob slides for some distance, the string becomes taut and the bob lifts off the floor. The magnitude of the angular momentum of the pendulum about the point of suspension just before the bob lifts off is

$J \text{ kg-m}^2/\text{s}$ . The kinetic energy of the pendulum just after the lift-off is  $K \text{ Joules}$ .

9. The value of  $J$  is \_\_\_\_\_.

**Ans. 0.18**

10. The value of  $K$  is \_\_\_\_\_.

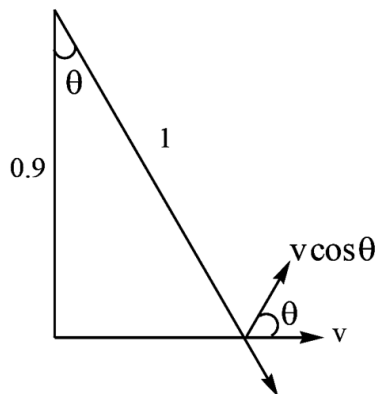
**Ans. 0.16**

## Sol. 9&10

$$P = mv \Rightarrow v = \frac{0.2}{0.1} = 2 \text{ m/s}$$

# MOMENTUM

$$|\vec{L}| = J = mvH = (0.9)(0.2) = 0.18$$



$$v \cos \theta = 1.8 \text{ ms}^{-1}$$

$$\therefore \text{K.E} = \frac{1}{2} m (1.8)^2 = 0.162$$

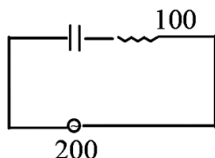
## Question Stem for Question Nos. 11 and 12

### Question Stem

In a circuit, a metal filament lamp is connected in series with a capacitor of capacitance  $C \mu\text{F}$  across a 200 V, 50 Hz supply. The power consumed by the lamp is 500 W while the voltage drop across it is 100 V. Assume that there is no inductive load in the circuit. Take *rms* values of the voltages. The magnitude of the phase-angle (in degrees) between the current and the supply voltage is  $\phi$ . Assume,  $\pi\sqrt{3} \approx 5$ .

11. The value of C is \_\_\_\_\_.

**Ans. 100**



**Sol.**

$$V_0^2 = V_R^2 + V_C^2$$

$$(200)^2 = (100)^2 + V_C^2$$

$$V_C = 100\sqrt{3}$$

$$P = (V)_R i_R$$

$$i(100) = 500$$

$$i = 5$$

$$i \times X_C = V_C$$

$$5 \frac{1}{2\pi(50)C} = 100\sqrt{3},$$

$$C = \frac{5}{(100\pi)100\sqrt{3}}$$

$$C = \frac{5}{10^4 \pi \sqrt{3}}$$

$$C = \frac{1}{10^4},$$

$$C = 100 \mu\text{F}$$

# MOMENTUM

12. The value of  $\phi$  is \_\_\_\_\_.

**Ans. 60**

**Sol.**  $\tan \phi = \frac{v_c}{v_R} = \frac{100\sqrt{3}}{100} \quad \phi = 60^\circ$

## SECTION-3(Maximum Marks: 12)

### Paragraph with Single Answer Type

- This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks : +3 If ONLY the correct option is chosen;*

*Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);*

*Negative Marks : -1 In all other cases.*

### Paragraph

A special metal S conducts electricity without any resistance. A closed wire loop, made of S, does not allow any change in flux through itself by inducing a suitable current to generate a compensating flux. The induced current in the loop cannot decay due to its zero resistance. This current gives rise to a magnetic moment which in turn repels the source of magnetic field or flux. Consider such a loop, of radius  $a$ , with its centre at the origin. A magnetic dipole of moment  $m$  is brought along the axis of this loop from infinity to a point at distance  $r$  ( $\gg a$ ) from the centre of the loop with its north pole always facing the loop, as shown in the figure below.

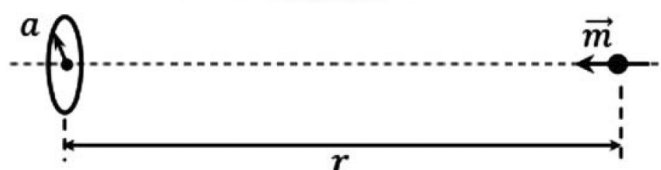
The magnitude of magnetic field of a dipole  $m$ , at a point on its axis at distance  $r$ , is

$\frac{\mu_0 m}{2\pi r^3}$ , where  $\mu_0$  is the permeability of free space. The magnitude of the force

between two magnetic dipoles with moments,  $m_1$  and  $m_2$ , separated by a distance

$r$  on the common axis, with their north poles facing each other, is  $\frac{k m_1 m_2}{r^4}$ , where  $k$

is a constant of appropriate dimensions. The direction of this force is along the line joining the two dipoles.



# MOMENTUM

13. When the dipole  $m$  is placed at a distance  $r$  from the centre of the loop (as shown in the figure), the current induced in the loop will be proportional to

A)  $m/r^3$                       B)  $m^2/r^2$                       C)  $m/r^2$                       D)  $m^2/r$

**Ans. A**

**Sol.** Suppose self-inductance of the loop =  $L$

$$\text{Then } \left( \frac{\mu_0 m}{2\pi r^3} \cdot \pi a^2 \right) - Li = 0 \quad \dots\dots\dots (1)$$

$$\Rightarrow i \propto \left( \frac{m}{r^3} \right)$$

14. The work done in bringing the dipole from infinity to a distance  $r$  from the centre of the loop by the given process is proportional to

A)  $m/r^5$                       B)  $m^2/r^5$                       C)  $m^2/r^6$                       D)  $m^2/r^7$

**Ans. C**

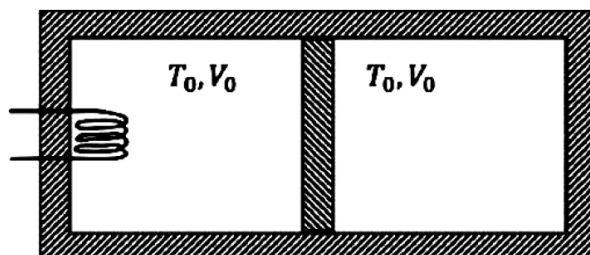
**Sol.**  $W = \int_{r=\infty}^r F dr$                       Also  $F = \frac{K m \cdot m_{\text{ind}}}{r^n}$

$$M_{\text{ind}} = \pi a^2 i = \pi a^2 \cdot \left( \frac{\mu_0 m}{2\pi r^3} \right) \frac{\pi a^2}{L} \quad (\text{from eqn. 1})$$

$$\therefore W = \int_{r=\infty}^r \frac{K m}{r^n} \cdot \frac{(\pi a^2)^2 \mu_0}{2\pi L} \cdot \frac{m}{r^3} \cdot dr \quad \propto \int_{r=\infty}^r \frac{m^2}{r^7} dr \quad \propto \frac{m^2}{r^6}$$

## Paragraph

A thermally insulating cylinder has a thermally insulating and frictionless movable partition in the middle, as shown in the figure below. On each side of the partition, there is one mole of an ideal gas, with specific heat at constant volume,  $C_V = 2R$ . Here,  $R$  is the gas constant. Initially, each side has a volume  $V_0$  and temperature  $T_0$ . The left side has an electric heater, which is turned on at very low power to transfer heat  $Q$  to the gas on the left side. As a result the partition moves slowly towards the right reducing the right side volume to  $V_0/2$ . Consequently, the gas temperatures on the left and the right sides become  $T_L$  and  $T_R$ , respectively. Ignore the changes in the temperatures of the cylinder, heater and the partition. N°/



# MOMENTUM

15. The value of  $\frac{T_R}{T_0}$  is

A)  $\sqrt{2}$

B)  $\sqrt{3}$

C) 2

D) 3

**Ans. A**

16. The value of  $\frac{Q}{RT_0}$  is

A)  $4(2\sqrt{2} + 1)$

B)  $4(2\sqrt{2} - 1)$

C)  $(5\sqrt{2} + 1)$

D)  $(5\sqrt{2} - 1)$

**Ans. B**

**Sol. 15&16**

Right chamber is undergoing adiabatic process.

$$\gamma = \frac{3}{2} \quad T_0 V_0^{\frac{3}{2}-1} = T_R \left( \frac{V_0}{2} \right)^{\frac{3}{2}-1}$$

$$T_R = \sqrt{2} T_0 \quad \therefore \frac{T_R}{T_0} = \sqrt{2}$$

$$P_f \left( \frac{3V_0}{2} \right) = 1RT_L$$

$$P_f \left( \frac{V_0}{2} \right) = 1RT_L \quad \therefore T_L = 3T_R$$

$$T_L = 3\sqrt{2}T_0$$

$$Q = (dU_L) + (dU)_R$$

$$Q = nC_v dT_L + nC_v dT_R$$

$$= 1(2R)(3\sqrt{2}T_0 - T_0) + 1(2R)(\sqrt{2}T_0 - T_0) = 2R(4\sqrt{2} - 2)T_0$$

$$Q = 4RT_0(2\sqrt{2} - 1)$$

$$\frac{Q}{RT_0} = 4(2\sqrt{2} - 1)$$

# MOMENTUM

## SECTION-4(Maximum Marks: 12)

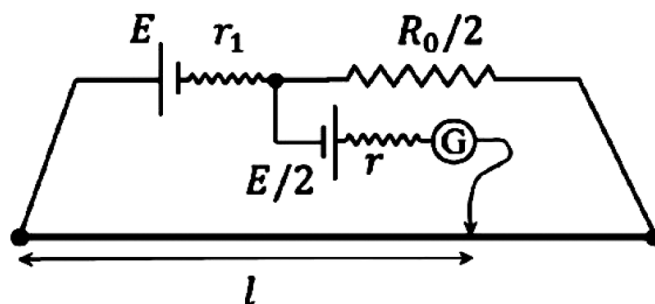
### Non-Negative Integer Answer Type

- This section contains THREE (03) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If ONLY the correct integer is entered;

Zero Marks : 0 In all other cases.

17. In order to measure the internal resistance  $r_1$  of a cell of emf  $E$ , a meter bridge of wire resistance  $R_0 = 50\Omega$ , a resistance  $R_0/2$ , another cell of emf  $E/2$  (internal resistance  $r$ ) and a galvanometer  $G$  are used in a circuit, as shown in the figure. If the null point is found at  $l = 72\text{ cm}$ , then the value of  $r_1 = \_\_\Omega$ .



Ans. 3

Sol.  $i$  in primary circuit  $= \frac{E}{R_0 + \frac{R_0}{2} + r_1}$

P.D between the points where the secondary cell is connected is

$$= i \left( \frac{R_0}{2} + \frac{28R_0}{100} \right) \quad \therefore \quad \frac{E}{2} = \frac{E}{\left( r_1 + \frac{3R_0}{2} \right)} \left( \frac{78R_0}{100} \right)$$

$$r_1 + \frac{3R_0}{2} = \frac{156R_0}{100}$$

$$r_1 = \frac{156 \times 50}{100} - \frac{3}{2} \times 50 = 78 - 75$$

$$r_1 = 78 - 75$$

$$r_1 = 3$$

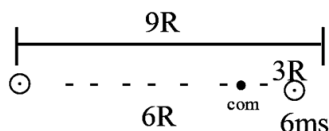
18. The distance between two stars of masses  $3M_S$  and  $6M_S$  is  $9R$ . Here  $R$  is the mean distance between the centres of the Earth and the Sun, and  $M_S$  is the mass of the Sun. The two stars orbit around their common centre of mass in circular orbits with period  $nT$ , where  $T$  is the period of Earth's revolution around the Sun. The value of  $n$  is \_\_\_\_.

# MOMENTUM

**Ans. 9**

**Sol.** 
$$\frac{G(3Ms)6Ms}{(9R)^2} = (6Ms)3R\omega^2$$

$$\omega = \sqrt{\frac{G Ms}{81R^3}} \quad T' = \frac{2\pi}{\omega}$$



$$T' = 2\pi \sqrt{\frac{81R^3}{GM}}$$

$$T' = 9T.$$

19. In a photoemission experiment, the maximum kinetic energies of photoelectrons from metals P, Q and R are  $E_P$ ,  $E_Q$  and  $E_R$ , respectively, and they are related by  $E_P = 2E_Q = 2E_R$ . In this experiment, the same source of monochromatic light is used for metals P and Q while a different source of monochromatic light is used for the metal R. The work functions for metals P, Q and R are 4.0 eV, 4.5 eV and 5.5 eV, respectively. The energy of the incident photon used for metal R, in eV, is \_\_\_\_.

**Ans. 6**

**Sol.**  $E_P = h\nu_1 - 4$

$$E_Q = h\nu_1 - 4.5$$

$$E_R = h\nu_2 - 5.5$$

$$E_P = 2E_Q = 2E_R$$

$$h\nu_1 - 4 = 2(h\nu_1 - 4.5)$$

$$9 - 4 = h\nu_1$$

$$E_Q = E_R$$

$$5 - 4.5 = h\nu_2 = 5.5$$

$$\Rightarrow h\nu_2 = 6\text{ eV}$$

$$E_{\text{incident photon}} = 6\text{ eV}$$

# MOMENTUM

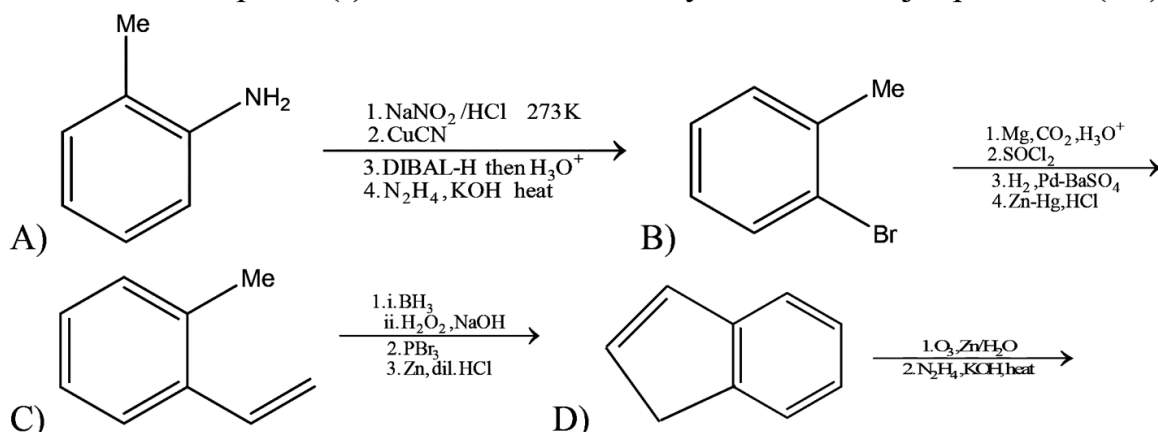
## PART-2: CHEMISTRY

### SECTION-1(Maximum Marks: 24)

#### One or More Type

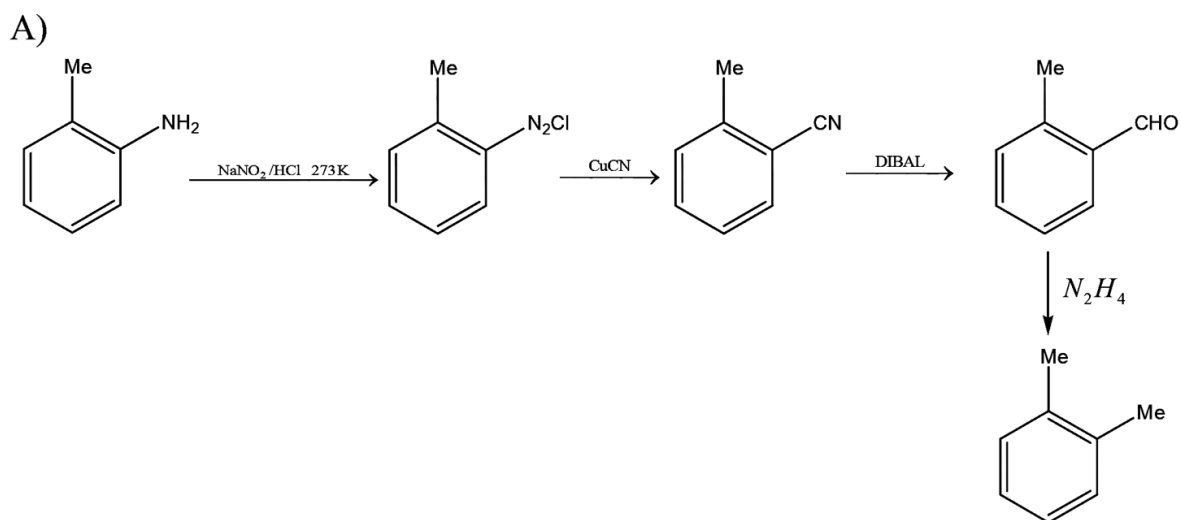
- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s)
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:  
 Full Marks : +4 If only (all) the correct option(s) is(are) chosen;  
 Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;  
 Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;  
 Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;  
 Zero Marks : 0 If unanswered;  
 Negative Marks : -2 In all other cases.

1. The reaction sequence(s) that would lead to *o*-xylene as the major product is(are)



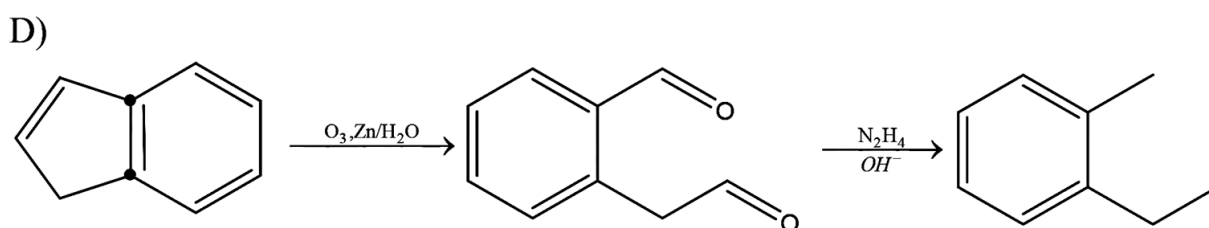
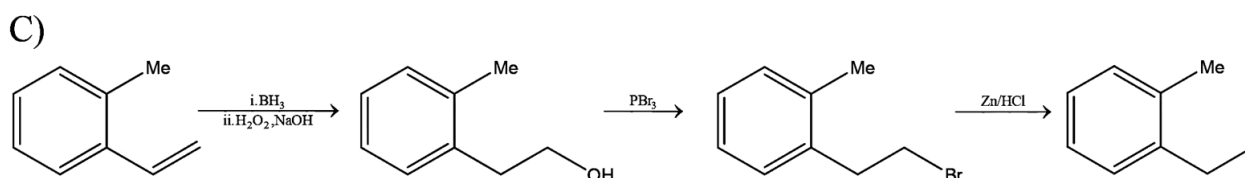
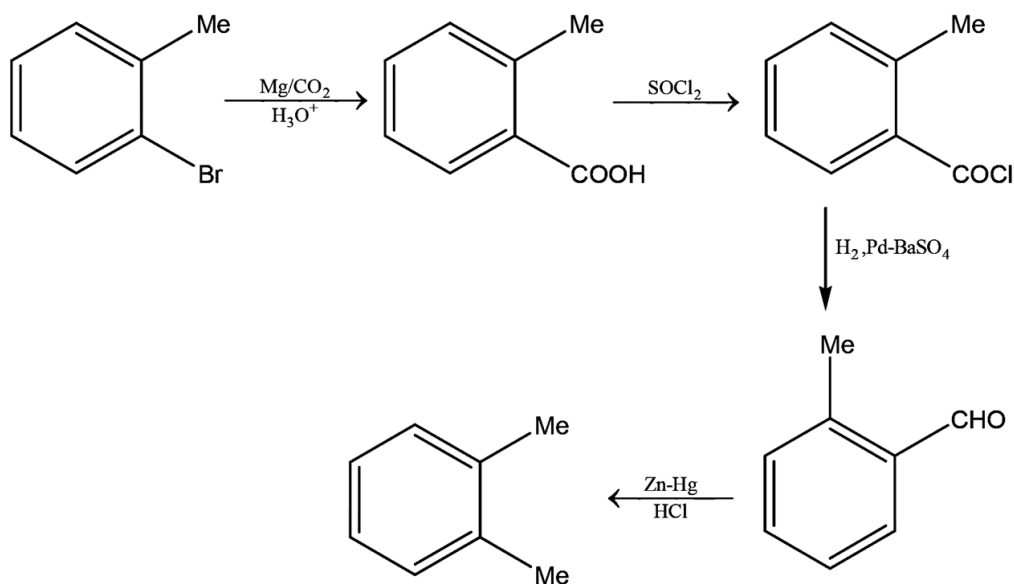
Ans : AB

Sol :

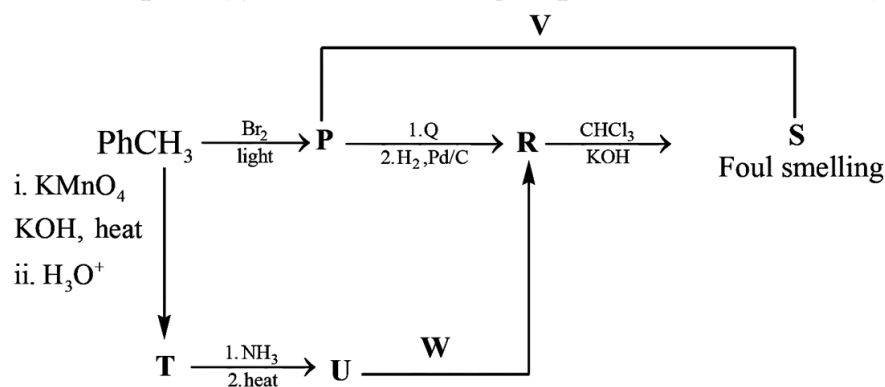


B)

# MOMENTUM



2. Correct option(s) for the following sequence of reactions is(are)

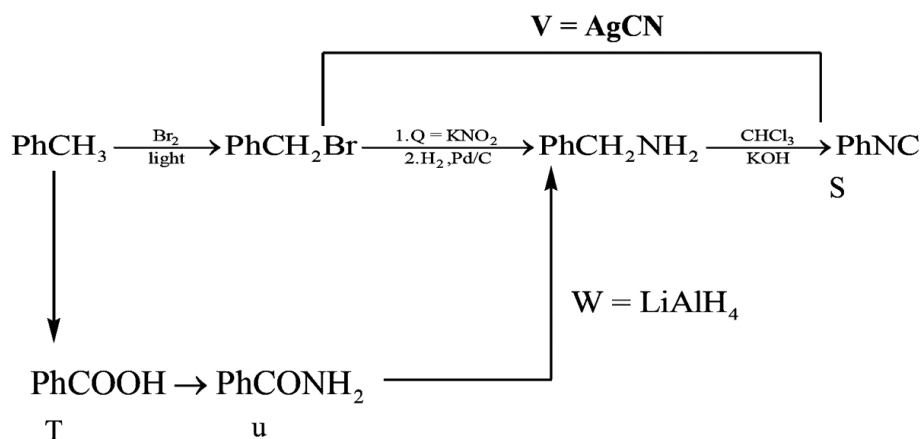


- A)  $\text{Q} = \text{KNO}_2$ ,  $\text{W} = \text{LiAlH}_4$   
 B)  $\text{R} = \text{benzenamine}$ ,  $\text{V} = \text{KCN}$   
 C)  $\text{Q} = \text{AgNO}_2$ ,  $\text{R} = \text{phenylmethanamine}$   
 D)  $\text{W} = \text{LiAlH}_4$ ,  $\text{V} = \text{AgCN}$

Ans : AD (As per standard ref) or CD (as per NCERT)

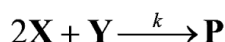
Sol :

# MOMENTUM



Note : As per the basic NCERT treatment, Q may be taken as  $AgNO_2$ . But as per mechanism,  $PhCH_2Br$  shown preferably react by  $S_N1$  mechanism. As  $Ag^+$  facilitate  $S_N1$  mechanism more,  $KNO_2$  should give more nitro products

3. For the following reaction



the rate of reaction is  $\frac{d[P]}{dt} = k[X]$ . Two moles of X are mixed with one mole of Y to make 1.0 L of solution. At 50 s, 0.5 mole of Y is left in the reaction mixture. The correct statement(s) about the reaction is(are)

(Use:  $\ln 2 = 0.693$ )

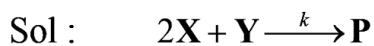
A) The rate constant,  $k$ , of the reaction is  $13.86 \times 10^{-4} s^{-1}$ .

B) Half-life of X is 50 s

C) At 50 s,  $-\frac{d[X]}{dt} = 13.86 \times 10^{-3} mol L^{-1} s^{-1}$ .

D) At 100 s,  $-\frac{d[Y]}{dt} = 3.46 \times 10^{-3} mol L^{-1} s^{-1}$ .

Ans : BCD



t = 0 2 mol 1 mol 0

t = 50 s (2 - 1) mol 0.5 mol 0.5 mol

$$k = \frac{1}{t} \ln \frac{a_0}{a_t} = \frac{1}{50} \ln \frac{2}{1}$$

$$= \frac{1}{50} \times 0.693$$

$$= 0.01386 s^{-1}$$

$$13.86 \times 10^{-3}$$

# MOMENTUM

$$\text{After } 100\text{s, } k = \frac{1}{100} \ln \frac{1}{a_t}$$

$$\frac{1}{50} \ln 2 = \frac{1}{100} \ln \frac{1}{a_t}$$

$$\therefore a_t = 0.25$$

$$\begin{aligned} \frac{d[p]}{dt} &= -\frac{d[y]}{dt} = (13.86 \times 10^{-3})(0.25) \\ &= 3.46 \times 10^{-3} \end{aligned}$$

4. Some standard electrode potentials at 298 K are given below:

$$\text{Pb}^{2+} / \text{Pb} \quad -0.13 \text{ V}$$

$$\text{Ni}^{2+} / \text{Ni} \quad -0.24 \text{ V}$$

$$\text{Cd}^{2+} / \text{Cd} \quad -0.40 \text{ V}$$

$$\text{Fe}^{2+} / \text{Fe} \quad -0.44 \text{ V}$$

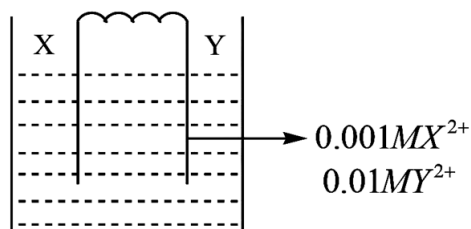
To a solution containing 0.001 M of  $\text{X}^{2+}$  and 0.1 M of  $\text{Y}^{2+}$ , the metal rods **X** and **Y** are inserted (at 298 K) and connected by a conducting wire. This resulted in dissolution of **X**. The correct combination(s) of **X** and **Y**, respectively, is(are)

(Given: Gas constant,  $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$ , Faraday constant,  $F = 96500 \text{ C mol}^{-1}$ )

A) Cd and Ni      B) Cd and Fe      C) Ni and Pb      D) Ni and Fe

Ans : ABC

Sol :



$$E_{\text{Pb}^{2+}/\text{Pb}} = E_{\text{Pb}^{2+}/\text{Pb}}^0 - 0.03 \log 10^3$$

$$= -0.13 - 0.03 \times 3$$

$$= -0.13 - 0.09$$

$$= -0.22 \text{ V}$$

$$E_{\text{Pb}^{2+}/\text{Pb}}^1 = -0.13 - 0.03 \log 10$$

$$= -0.13 - 0.03$$

$$= -0.16 \text{ V}$$

$$E_{\text{Ni}^{2+}/\text{Ni}} = -0.24 - 0.09 = -0.33 \text{ V}$$

$$E_{\text{Ni}^{2+}/\text{Ni}}^1 = -0.24 - 0.03 = -0.27$$

$$E_{\text{Cd}^{2+}/\text{Cd}} = -0.40 - 0.09 = -0.49 \text{ V}$$

# MOMENTUM

$$E_{Cd^{2+}/Cd}^1 = -0.40 - 0.03 = -0.43V$$

$$E_{Fe^{2+}/Fe} = -0.40 - 0.09 = -0.53V$$

$$E_{Fe^{2+}/Fe}^1 = -0.44 - 0.03 = -0.47V$$

$$\Delta G = -nFE_{cell}$$

$$\Delta G \text{ of } A = -ve$$

$$\Delta G \text{ of } B = -ve$$

$$\Delta G \text{ of } C = -ve$$

$$\Delta G \text{ of } D = +ve$$

5. The pair(s) of complexes where in both exhibit tetrahedral geometry is(are)

(Note: py = pyridine)

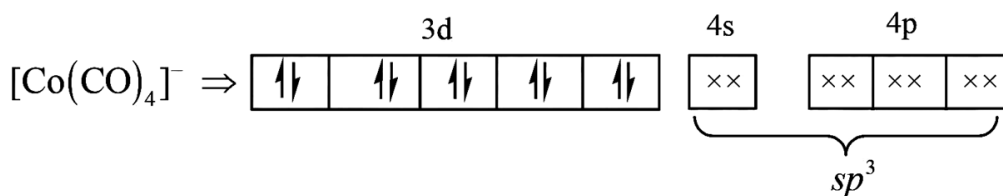
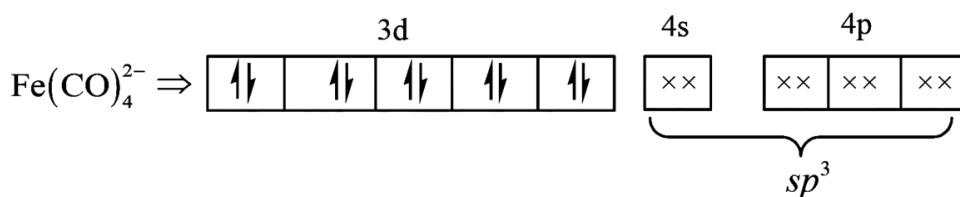
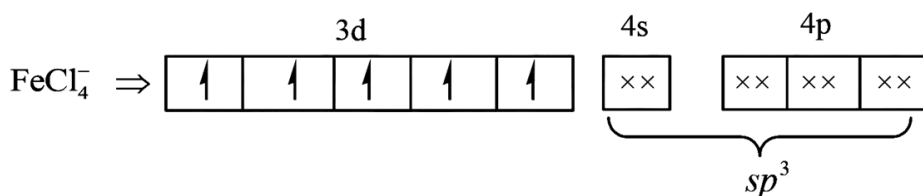
Given: Atomic numbers of Fe, Co, Ni and Cu are 26, 27, 28 and 29, respectively)

- A)  $[FeCl_4]^-$  and  $[Fe(CO)_4]^{2-}$       B)  $[Co(CO)_4]^-$  and  $[CoCl_4]^{2-}$   
 C)  $[Ni(CO)_4]$  and  $[Ni(CN)_4]^{2-}$       D)  $[Cu(py)_4]^+$  and  $[Cu(CN)_4]^{3-}$

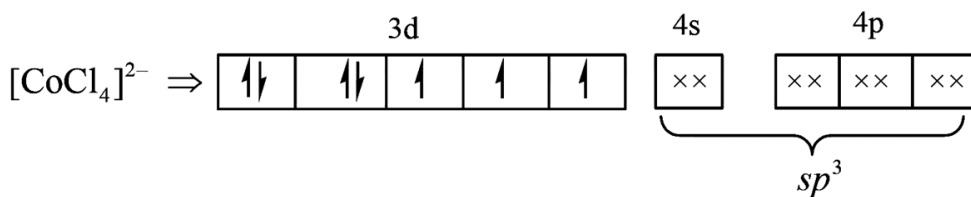
Ans : ABD

Sol : A)  $FeCl_4^-$

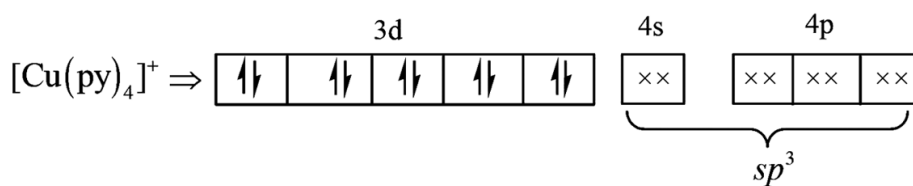
$$Fe^{2+} = 3d^6 4s^0$$



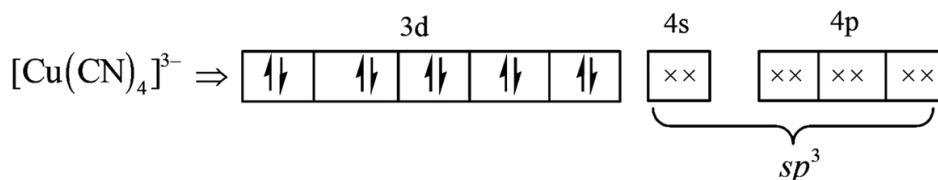
B)



# MOMENTUM



D)



6. The correct statement(s) related to oxoacids of phosphorous is(are)
- A) Upon heating,  $\text{H}_3\text{PO}_3$  undergoes disproportionation reaction to produce  $\text{H}_3\text{PO}_4$  and  $\text{PH}_3$ .
- B) While  $\text{H}_3\text{PO}_3$  can act as reducing agent,  $\text{H}_3\text{PO}_4$  cannot.
- C)  $\text{H}_3\text{PO}_3$  is a monobasic acid.
- D) The H atom of P–H bond in  $\text{H}_3\text{PO}_3$  is not ionizable in water.

Ans : ABD

Sol : A)  $4\text{H}_3\text{PO}_3 \rightarrow 3\text{H}_3\text{PO}_4 + \text{PH}_3$

B)  $\text{H}_3\text{PO}_3$  is a reducing acid as it has P – H bond

C)  $\text{H}_3\text{PO}_3$  is a dibasic acid

D)  $\text{H}_3\text{PO}_3 + \text{H}_2\text{O} \rightleftharpoons \text{H}_3\text{PO}_3^- + \text{H}_3\text{O}^+$

$\text{H}_3\text{PO}_3^- + \text{H}_2\text{O} \rightleftharpoons \text{H}_3\text{PO}_3^{2-} + \text{H}_3\text{O}^+$

## SECTION-2(Maximum Marks: 12)

### Paragraph with Numerical

- This section contains THREE (03) question stems.
- There are TWO (02) questions corresponding to each question stem.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:  
Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;  
Zero Marks : 0 In all other cases.

### Question Stem for Question Nos. 7 and 8

At 298 K, the limiting molar conductivity of a weak monobasic acid is  $4 \times 10^2 \text{ S cm}^2 \text{ mol}^{-1}$ . At 298 K, for an aqueous solution of the acid the degree of dissociation is  $\alpha$  and the molar conductivity is  $y \times 10^2 \text{ S cm}^2 \text{ mol}^{-1}$ . At 298 K, upon

# MOMENTUM

20 times dilution with water, the molar conductivity of the solution becomes  $3y \times 10^2 \text{ S cm}^2 \text{ mol}^{-1}$ .

7. The value of  $\alpha$  is \_\_\_\_.

Ans : 0.22

Sol :  $\wedge^\circ = 4 \times 10^2 \text{ S cm}^2 \text{ mol}^{-1}$

For concentration C,

$$\alpha = \frac{y \times 10^2}{4 \times 10^2} = \frac{y}{4}$$

$$k = \frac{C \cdot \alpha^2}{1 - \alpha} = \frac{C \left( \frac{y}{4} \right)^2}{1 - \frac{y}{4}}$$

For concentration  $\frac{C}{20}$ ,

$$k = \frac{\frac{C}{20} \cdot \left( \frac{3y}{4} \right)^2}{1 - \frac{3y}{4}}$$

$$\Rightarrow y = \frac{44}{51} = 0.86$$

$$\alpha = \frac{11}{51} = 0.21 \text{ to } 0.22$$

8. The value of y is \_\_\_\_.

Ans : 0.86

Sol :  $\wedge^\circ = 4 \times 10^2 \text{ S cm}^2 \text{ mol}^{-1}$

For concentration C,

$$\alpha = \frac{y \times 10^2}{4 \times 10^2} = \frac{y}{4}$$

$$k = \frac{C \cdot \alpha^2}{1 - \alpha} = \frac{C \left( \frac{y}{4} \right)^2}{1 - \frac{y}{4}}$$

For concentration  $\frac{C}{20}$ ,

$$k = \frac{\frac{C}{20} \cdot \left( \frac{3y}{4} \right)^2}{1 - \frac{3y}{4}} \Rightarrow y = \frac{44}{51} = 0.86$$

$$\alpha = \frac{11}{51} = 0.21 \text{ to } 0.22$$

# MOMENTUM

## Question Stem for Question Nos. 9 and 10

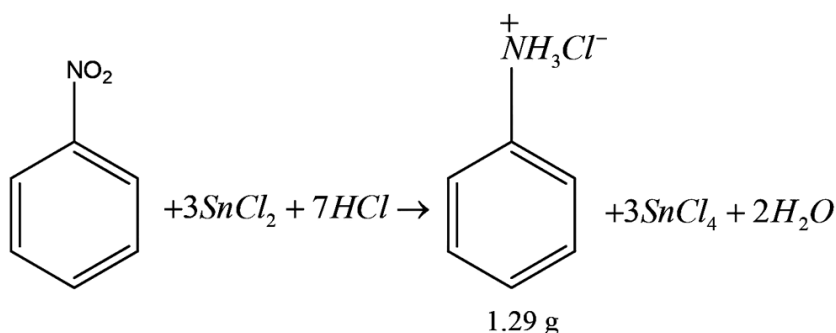
Reaction of x g of Sn with HCl quantitatively produced a salt. Entire amount of the salt reacted with y g of nitrobenzene in the presence of required amount of HCl to produce 1.29 g of an organic salt (quantitatively).

(Use Molar masses (in  $\text{g mol}^{-1}$ ) of H, C, N, O, Cl and Sn as 1, 12, 14, 16, 35 and 119, respectively).

9. The value of x is \_\_\_\_.

Ans : 3.57

Sol :  $\text{Sn} + 2\text{HCl} \rightarrow \text{SnCl}_2 + \text{H}_2$



$$n_{\text{PhNH}_3^+\text{Cl}^-} = \frac{1.29}{129} = 0.01$$

$$\therefore n_{\text{SnCl}_2} = 0.03$$

$$\therefore n_{\text{Sn}} = 0.03$$

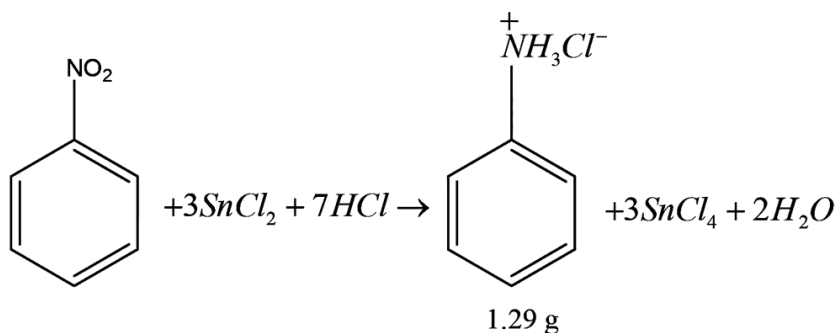
$$x = 0.03 \times 114 = 3.57$$

$$y = 0.01 \times 123 = 1.23$$

10. The value of y is \_\_\_\_.

Ans : 1.23

Sol :  $\text{Sn} + 2\text{HCl} \rightarrow \text{SnCl}_2 + \text{H}_2$



$$n_{\text{PhNH}_3^+\text{Cl}^-} = \frac{1.29}{129} = 0.01$$

$$\therefore n_{\text{SnCl}_2} = 0.03$$

$$\therefore n_{\text{Sn}} = 0.03$$

$$x = 0.03 \times 114 = 3.57$$

$$y = 0.01 \times 123 = 1.23$$

# MOMENTUM

## Question Stem for Question Nos. 11 and 12

A sample (5.6 g) containing iron is completely dissolved in cold dilute HCl to prepare a 250 mL of solution. Titration of 25.0 mL of this solution requires 12.5 mL of 0.03 M  $\text{KMnO}_4$  solution to reach the end point. Number of moles of  $\text{Fe}^{2+}$  present in 250 mL solution is  $x \times 10^{-2}$  (consider complete dissolution of  $\text{FeCl}_2$ ). The amount of iron present in the sample is  $y\%$  by weight.

(Assume:  $\text{KMnO}_4$  reacts only with  $\text{Fe}^{2+}$  in the solution)

Use: Molar mass of iron as  $56 \text{ g mol}^{-1}$ )

11. The value of  $x$  is \_\_\_\_.

Ans : 1.875

Sol :  $n_{\text{gl of Fe}^{2+}} = n_{\text{ge KMnO}_4}$

$$= 12.5 \times 10^{-3} \times 0.03 \times 5$$

$$= 1.875 \times 10^{-3}$$

$$\therefore n_{\text{Fe}^{2+}} \text{ in 250 of } SA_n$$

$$= 1.875 \times 10^{-3} \times 10$$

$$= 1.875 \times 10^{-2}$$

$$\Rightarrow x = 1.875$$

$$W_{\text{Fe}} = 1.875 \times 10^{-2} \times 56$$

$$= 1.05 \text{ g}$$

$$\text{Fe} = \frac{1.05}{5.6} \times 100 = 18.75$$

12. The value of  $y$  is \_\_\_\_.

Ans : 18.75

Sol :  $n_{\text{gl of Fe}^{2+}} = n_{\text{ge KMnO}_4}$

$$= 12.5 \times 10^{-3} \times 0.03 \times 5$$

$$= 1.875 \times 10^{-3}$$

$$\therefore n_{\text{Fe}^{2+}} \text{ in 250 of } SA_n$$

$$= 1.875 \times 10^{-3} \times 10$$

$$= 1.875 \times 10^{-2} \quad \Rightarrow x = 1.875$$

$$W_{\text{Fe}} = 1.875 \times 10^{-2} \times 56$$

$$= 1.05 \text{ g}$$

$$\text{Fe} = \frac{1.05}{5.6} \times 100 = 18.75$$

# MOMENTUM

## SECTION-3(Maximum Marks: 12)

### Paragraph with Single Answer Type

- This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer
- Answer to each question will be evaluated according to the following marking scheme:

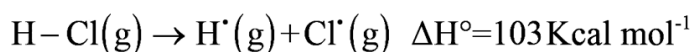
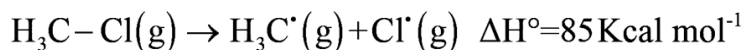
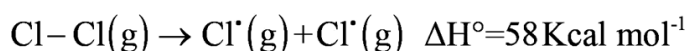
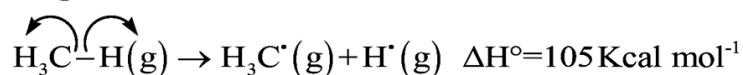
Full Marks : +3 If ONLY the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : -1 In all other cases.

### Paragraph-1:

The amount of energy required to break a bond is same as the amount of energy released when the same bond is formed. In gaseous state, the energy required for homolytic cleavage of a bond is called Bond Dissociation Energy (BDE) or Bond Strength. BDE is affected by s-character of the bond and the stability of the radicals formed. Shorter bonds are typically stronger bonds. BDEs for some bonds are given below:



13. Correct match of the **C-H** bonds (shown in bold) in Column **J** with their BDE in Column **K** is

Column <b>J</b> Molecule	Column <b>K</b> BDE (kcal mol <sup>-1</sup> )
P) <b>H</b> - CH(CH <sub>3</sub> ) <sub>2</sub>	i) 132
Q) <b>H</b> - CH <sub>2</sub> Ph	ii) 110
R) <b>H</b> - CH = CH <sub>2</sub>	iii) 95
S) <b>H</b> - C ≡ CH	iv) 88

A) P - iii, Q - iv, R - ii, S - i

B) P - i, Q - ii, R - iii, S - iv

C) P - iii, Q - ii, R - i, S - iv

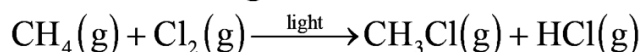
D) P - ii, Q - i, R - iv, S - iii

Ans : A

Sol : Q < P < R < S

# MOMENTUM

14. For the following reaction



the correct statement is

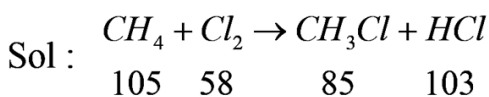
A) Initiation step is exothermic with  $\Delta H^\circ = -58 \text{ kcal mol}^{-1}$

B) Propagation step involving  $\cdot\text{CH}_3$  formation is exothermic with  $\Delta H^\circ = -2 \text{ kcal mol}^{-1}$

C) Propagation step involving  $\text{CH}_3\text{Cl}$  formation is endothermic with  $\Delta H^\circ = +27 \text{ kcal mol}^{-1}$ .

D) The reaction is exothermic with  $\Delta H^\circ = -25 \text{ kcal mol}^{-1}$ .

Ans : D



$$\Delta H = 105 + 58 - (85 + 103) = -25 \text{ kcal mol}^{-1}$$

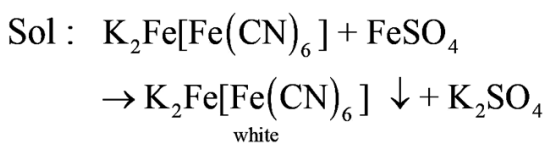
## Paragraph-2:

The reaction of  $\text{K}_3[\text{Fe}(\text{CN})_6]$  with freshly prepared  $\text{FeSO}_4$  solution produces a dark blue precipitate called Turnbull's blue. Reaction of  $\text{K}_4[\text{Fe}(\text{CN})_6]$  with the  $\text{FeSO}_4$  solution in complete absence of air produces a white precipitate X, which turns blue in air. Mixing the  $\text{FeSO}_4$  solution with  $\text{NaNO}_3$ , followed by a slow addition of concentrated  $\text{H}_2\text{SO}_4$  through the side of the test tube produces a brown ring.

15. Precipitate X is

A)  $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3$  B)  $\text{Fe}[\text{Fe}(\text{CN})_6]$  C)  $\text{K}_2\text{Fe}[\text{Fe}(\text{CN})_6]$  D)  $\text{KFe}[\text{Fe}(\text{CN})_6]$

Ans : C



16. Among the following, the brown ring is due to the formation of

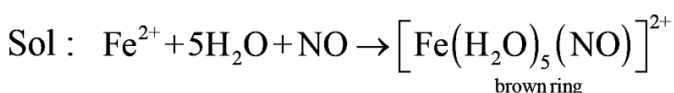
A)  $[\text{Fe}(\text{NO})_2(\text{SO}_4)_2]^{2-}$

B)  $[\text{Fe}(\text{NO})_2(\text{H}_2\text{O})_4]^{3+}$

C)  $[\text{Fe}(\text{NO})_4(\text{SO}_4)_2]$

D)  $[\text{Fe}(\text{NO})(\text{H}_2\text{O})_5]^{2+}$

Ans : D



# MOMENTUM

## SECTION-4(Maximum Marks: 12)

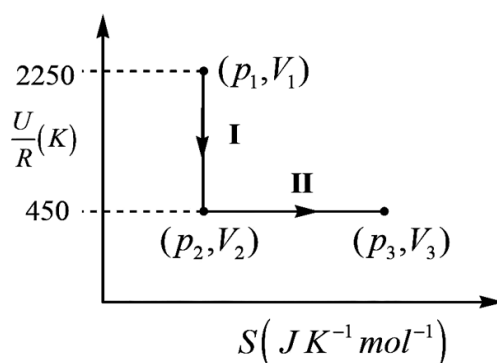
### Non-Negative Integer Answer Type

- This section contains THREE (03) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If ONLY the correct integer is entered;

Zero Marks : 0 In all other cases.

17. One mole of an ideal gas at 900 K, undergoes two reversible processes, I followed by II, as shown below. If the work done by the gas in the two processes are same, the value of  $\ln \frac{V_3}{V_2}$  is \_\_\_\_.



( $U$ : internal energy,  $S$ : entropy,  $p$ : pressure,  $V$ : volume,  $R$ : gas constant)

(Given: molar heat capacity at constant volume,  $C_{V,m}$  of the gas is  $\frac{5}{2}R$ )

Ans : 10

Sol :  $\frac{\Delta u}{R} = 1800 \Rightarrow \Delta u = 1800 \times R = W$

$$1800R = nRT \ln \frac{V_3}{V_2}$$

For process I

$$Q = 0$$

$$\Delta u = W = 1800 R$$

For process II

$$\Delta u = 0$$

$$u = W = nRT \ln \frac{V_3}{V_2}$$

$$T_1 = 900 K$$

# MOMENTUM

$$\Delta u = nC_v \Delta T \quad -1800R = 1 \times \frac{5R}{2}(T - 900)$$

$$T = 180 \text{ K}$$

$$1 \times 180 \ln \frac{V_3}{V_2} = 1800$$

$$\ln \frac{V_3}{V_2} = 10$$

18. Consider a helium (He) atom that absorbs a photon of wavelength 330 nm. The change in the velocity (in  $\text{cm s}^{-1}$ ) of He atom after the photon absorption is \_\_\_\_.

(Assume: Momentum is conserved when photon is absorbed.)

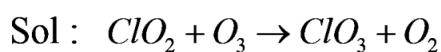
Use: Planck constant =  $6.6 \times 10^{-34} \text{ J s}$ , Avogadro number =  $6 \times 10^{23} \text{ mol}^{-1}$ , Molar mass of He =  $4 \text{ g mol}^{-1}$ )

Ans : 30

$$\text{Sol : } V = \frac{h}{m\lambda} = \frac{6.6 \times 10^{-34} \times 6 \times 10^{23}}{4 \times 10^{-3} \times 330 \times 10^{-9}} \text{ cm s}^{-1}$$

19. Ozonolysis of  $\text{ClO}_2$  produces an oxide of chlorine. The average oxidation state of chlorine in this oxide is \_\_\_\_.

Ans : 6



$\therefore$  Oxidation state of Cl in  $\text{ClO}_3$  is

# MOMENTUM

## PART-3: MATH

### SECTION-1(Maximum Marks: 24)

#### One or More Type

- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s)
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:  
Full Marks : +4 If only (all) the correct option(s) is(are) chosen;  
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;  
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;  
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;  
Zero Marks : 0 If unanswered;  
Negative Marks : -2 In all other cases.

1. Let

$$S_1 = \{(i, j, k) : i, j, k \in \{1, 2, \dots, 10\}\},$$

$$S_2 = \{(i, j) : 1 \leq i < j + 2 \leq 10, i, j \in \{1, 2, \dots, 10\}\},$$

$$S_3 = \{(i, j, k, l) : 1 \leq i < j < k < l, i, j, k, l \in \{1, 2, \dots, 10\}\}$$

and

$$S_4 = \{(i, j, k, l) : i, j, k \text{ and } l \text{ are distinct elements in } \{1, 2, \dots, 10\}\}.$$

If the total number elements in the set  $S_r$  is  $n_r, r = 1, 2, 3, 4$ , then which of the following statements is(are) TRUE?

A)  $n_1 = 1000$       B)  $n_2 = 44$       C)  $n_3 = 220$       D)  $\frac{n_4}{12} = 420$

Ans: ABD

Sol:  $n_1 = 10 \times 10 \times 10$

$$n_2 = 8 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 8 + \frac{8 \times 9}{2} = 44$$

$$[i = 1, j = 1, 2, 3, \dots, 8]; [i = 2, j = 1, 2, 3, \dots, 8]; [i = 3, j = 2, 3, \dots, 8], \dots \text{ and so on}$$

$$n_3 = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{24} = 210 \text{ (Select 4 numbers and arrange in increasing order)}$$

$$n_4 = {}^{10}P_4 = 10 \times 9 \times 8 \times 7 \Rightarrow \frac{n_4}{12} = 420$$

# MOMENTUM

2. Consider a triangle PQR having sides of lengths p, q and r opposite to the angles P, Q and R, respectively. Then which of the following statements is(are) TRUE?

A)  $\cos P \geq 1 - \frac{p^2}{2qr}$

B)  $\cos R \geq \left(\frac{q-r}{p+q}\right) \cos P + \left(\frac{p-r}{p+q}\right) \cos Q$

C)  $\frac{q+r}{p} < 2 \frac{\sqrt{\sin Q \sin R}}{\sin P}$

D) If  $p < q$  and  $p < r$ , then  $\cos Q > \frac{p}{r}$  and  $\cos R > \frac{p}{q}$

Ans: AB

Sol: A)  $\cos P = \frac{q^2 + r^2}{2qr} - \frac{p^2}{2qr} \geq 1 - \frac{p^2}{2qr}$

B)  $p + q > r \Rightarrow (r \cos Q + q \cos R) + (p \cos R + r \cos P) > (p \cos Q + q \cos P)$

C)  $\frac{q+r}{P} = \frac{\sin Q + \sin R}{\sin P} \geq 2 \frac{\sqrt{\sin Q \cdot \sin R}}{\sin P}$

D)  $\cos Q > \frac{q}{r} \Rightarrow \angle Q$  is acute  $\Rightarrow \angle R$  is acute (similarly)

$\cos Q > \frac{q}{r} \Rightarrow \frac{p^2 - q^2 + r^2}{2pr} > \frac{q}{r} \Rightarrow p^2 + r^2 - q^2 > 2p^2 \Rightarrow p^2 + q^2 < r^2$

$\angle R$  is obtuse (contradiction)

3. Let  $f: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow \mathbb{R}$  be a continuous function such that

$f(0) = 1$  and  $\int_0^{\pi/3} f(t) dt = 0$

Then which of the following statements is(are) TRUE?

A) The equation  $f(x) - 3 \cos 3x = 0$  has at least one solution in  $\left(0, \frac{\pi}{3}\right)$

B) The equation  $f(x) - 3 \sin 3x = -\frac{6}{\pi}$  has at least one solution in  $\left(0, \frac{\pi}{3}\right)$

C)  $\lim_{x \rightarrow 0} \frac{\int_0^x f(t) dt}{1 - e^{x^2}} = -1$

D)  $\lim_{x \rightarrow 0} \frac{\sin x \int_0^x f(t) dt}{x^2} = -1$

# MOMENTUM

Ans: ABC

Sol:  $f(0) = 1, \int_0^{\pi/3} f(t) dt = 0$

A) Let  $g(x) = \int_0^x f(t) dt - \sin 3x$

$g(0) = 0 = g(\pi/3) \Rightarrow g'(x) = 0$  has atleast one solution in  $(0, \pi/3)$

B) Let  $g(x) = \int_0^x f(t) dt + \cos 3x + \frac{6x}{\pi}$

$g(0) = 1 = g(\pi/3)$

$\Rightarrow g'(x) = 0$  has atleast one solution in  $(0, \pi/3)$

C)  $\lim_{x \rightarrow 0} \frac{x \int_0^x f(t) dt}{1 - e^{x^2}} = \lim_{x \rightarrow 0} \frac{\int_0^x f(t) dt + xf(x)}{-2xe^{x^2}}$   
 $= \lim_{x \rightarrow 0} \frac{f(x) + xf'(x) + f(x)}{-2[e^{x^2} + 2x^2ex^2]} = -1$

D)  $\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) \left( \frac{\int_0^x f(t) dt}{x} \right) = 1 \times 1 = +1$

4. For any real numbers  $\alpha$  and  $\beta$ , let  $y_{\alpha, \beta}(x), x \in \mathbb{R}$ , be the solution of the differential equation  $\frac{dy}{dx} + \alpha y = xe^{\beta x}, y(1) = 1$ .

Let  $S = \{y_{\alpha, \beta}(x) : \alpha, \beta \in \mathbb{R}\}$ . Then which of the following functions belong(s) to the set S?

A)  $f(x) = \frac{x^2}{2}e^{-x} + \left(e - \frac{1}{2}\right)e^{-x}$       B)  $f(x) = -\frac{x^2}{2}e^{-x} + \left(e + \frac{1}{2}\right)e^{-x}$

C)  $f(x) = \frac{e^x}{2}\left(x - \frac{1}{2}\right) + \left(e - \frac{e^2}{4}\right)e^{-x}$       D)  $f(x) = \frac{e^x}{2}\left(\frac{1}{2} - x\right) + \left(e + \frac{e^2}{4}\right)e^{-x}$

Ans: AC

Sol:  $ye^{\alpha x} = \int x e^{(\alpha+\beta)x} dx$

If  $\alpha + \beta = 0$ , then  $ye^{\alpha x} = \frac{x^2}{2} + c$

# MOMENTUM

$$(1,1) \Rightarrow e^\alpha = \frac{1}{2} + c \Rightarrow c = e^\alpha - \frac{1}{2}$$

$$\therefore y = \left( \frac{x^2}{2} + e^\alpha - \frac{1}{2} \right) e^{-\alpha x}$$

$$\text{Put } \alpha = 1 \Rightarrow y = \left( \frac{x^2}{2} + e - \frac{1}{2} \right) e^{-2} \quad (\text{A is correct})$$

$$\text{If } \alpha + \beta = 0, \text{ then } ye^{\alpha x} = \frac{xe^{(\alpha+\beta)x}}{\alpha + \beta} - \frac{e^{(\alpha+\beta)x}}{(\alpha + \beta)^2} + c$$

$$\text{Put } \alpha = \beta = 1$$

$$y = \frac{x}{2} e^x - \frac{e^x}{4} + c$$

$$(1,1) \Rightarrow y = \frac{e^x}{2} \left( x - \frac{1}{2} \right) + e^{-x} \left( e - \frac{e^2}{4} \right) \quad (\text{C is correct})$$

5. Let O be the origin and  $\vec{OA} = 2\hat{i} + 2\hat{j} + \hat{k}$ ,  $\vec{OB} = \hat{i} - 2\hat{j} + 2\hat{k}$  and  $\vec{OC} = \frac{1}{2}(\vec{OB} - \lambda\vec{OA})$  for

some  $\lambda > 0$ . If  $|\vec{OB} \times \vec{OC}| = \frac{9}{2}$ , then which of the following statements is(are) True?

A) Projection of  $\vec{OC}$  on  $\vec{OA}$  is  $-\frac{3}{2}$

B) Area of the triangle OAB is  $\frac{9}{2}$

C) Area of the triangle ABC is  $\frac{9}{2}$

D) The acute angle between the diagonals of the parallelogram with adjacent sides  $\vec{OA}$  and  $\vec{OC}$  is  $\frac{\pi}{3}$

Ans: ABC

$$\text{Sol: } \vec{OC} = \frac{1}{2}[(1-2\lambda)\hat{i} - (2)(1+\lambda)\hat{j} + (2-\lambda)\hat{k}]$$

$$|\vec{OB} \times \vec{OC}| = \frac{9}{2} = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 1-2\lambda & -2-2\lambda & 2-\lambda \end{vmatrix} = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2\lambda-1 & 2\lambda+2 & \lambda-2 \end{vmatrix}$$

$$= \frac{1}{2} |-6\lambda\hat{i} + 3\lambda\hat{j} + (6\lambda)\hat{k}|$$

$$\Rightarrow 9 = \sqrt{36 + 9 + 36} |\lambda| = \lambda = 1$$

# MOMENTUM

$$\therefore \vec{OC} = \frac{1}{2}(-\hat{i} - 4\hat{j} + \hat{k})$$

$$A) \frac{\vec{OC} \cdot \vec{OA}}{OA} = \frac{-2 - 8 + 1}{2.3} = \frac{-3}{2}$$

$$B) \text{ar}(\Delta OAB) = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{vmatrix} = \frac{1}{2} |-6\hat{i} - 3\hat{j} - 6\hat{k}| = \frac{9}{2}$$

$$C) \text{ar}(ABC) = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -1 \\ \frac{5}{2} & \frac{8}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{4} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -1 \\ 5 & 8 & 1 \end{vmatrix}$$

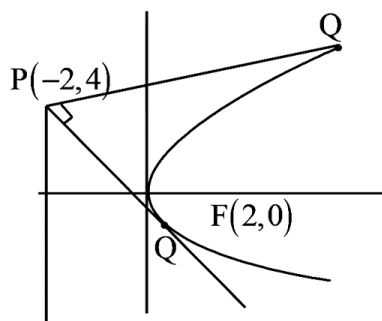
$$= \frac{1}{4} |12\hat{i} - 6\hat{j} - 12\hat{k}| = \frac{1}{2} |6\hat{i} - 3\hat{j} - 6\hat{k}| = \frac{9}{2}$$

$$D) \vec{d_1} = \vec{OA} + \vec{OC}, \vec{d_2} = \vec{OA} - \vec{OC} \quad \cos\theta = \frac{\vec{d_1} \cdot \vec{d_2}}{|\vec{d_1}| \cdot |\vec{d_2}|} = \frac{9 - \frac{9}{2}}{\frac{3}{2}\sqrt{2} \cdot \frac{3\sqrt{10}}{2}} = \sqrt{\frac{2}{5}}$$

6. Let E denote the parabola  $y^2 = 8x$ . Let  $P = (-2, 4)$ , and let Q and Q' be two distinct points on E such that the lines PQ and PQ' are tangents to E. Let F be the focus of E. Then which of the following statements is(are) TRUE?
- A) The triangle PFQ is a right-angled triangle
  - B) The triangle QPQ' is a right-angled triangle
  - C) The distance between P and F is  $5\sqrt{2}$
  - D) F lies on the line joining Q and Q'

Ans: ABD

Sol:  $y^2 = 8x$ ;  $P(-2, 4)$  lies on directrix



- A) Portion of tangent between POC and directrix subtends  $90^\circ$  at focus
- B) Tangents at ends of focal chords are perpendicular
- C)  $\sqrt{4+4} = 4\sqrt{2}$
- D) QQ' is a focal chord.

# MOMENTUM

## SECTION-2(Maximum Marks: 12)

### Paragraph with Numerical

- This section contains THREE (03) question stems.
- There are TWO (02) questions corresponding to each question stem.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;

Zero Marks : 0 In all other cases.

### Question Stem for Question Nos. 7 and 8

Consider the region  $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x \geq 0 \text{ and } y^2 \leq 4 - x\}$ . Let  $F$  be the family of all circles that are contained in  $R$  and have centers on the  $x$ -axis. Let  $C$  be the circle that has largest radius among the circle in  $F$ . Let  $(\alpha, \beta)$  be a point where the circle  $C$  meets the curve  $y^2 = 4 - x$ .

7. The radius of the circle  $C$  is \_\_\_\_\_.

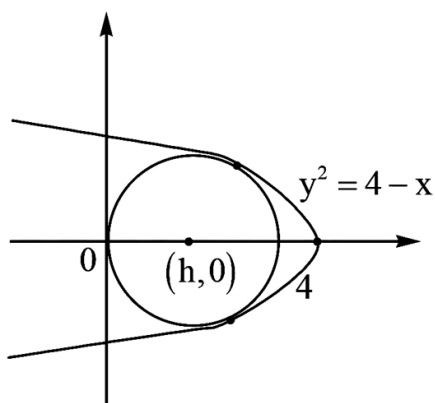
Ans: 1.5

8. The value of  $\alpha$  is \_\_\_\_\_.

Ans: 2

**Sol(7&8Q):**

$$x \geq 0, y^2 \leq 4 - x$$



Let equation of circle be  $(x - h)^2 + y^2 = h^2$

Solving with  $y^2 = 4 - x$

$$x^2 - 2hx + 4 - x = 0$$

$$\Rightarrow x^2 - x(2h + 1) + 4 = 0 \quad \dots(1)$$

For touching,  $D = 0$

$$\Rightarrow (2h + 1)^2 = 16$$

# MOMENTUM

$$\Rightarrow 2h + 1 = 4 \quad \Rightarrow h = \frac{3}{2}$$

Putting  $h = \frac{3}{2}$  in (1)

$$x^2 - 4x + 4 = 0 \quad \Rightarrow x = 2$$

So  $\alpha = 2$

## Question Stem for Question Nos. 9 and 10

Let  $f_1 : (0, \infty) \rightarrow \mathbb{R}$  and  $f_2 : (0, \infty) \rightarrow \mathbb{R}$  be defined by

$$f_1(x) = \int_0^x \prod_{j=1}^{21} (t-j)^j dt, \quad x > 0$$

and

$$f_2(x) = 98(x-1)^{50} - 600(x-1)^{49} + 2450, x > 0,$$

where, for any positive integer  $n$  and real numbers  $a_1, a_2, \dots, a_n$ ,  $\prod_{i=1}^n a_i$  denotes the product of  $a_1, a_2, \dots, a_n$ . Let  $m_i$  and  $n_i$ , respectively, denote the number of points of local minima and the number of points of local maxima of function  $f_i, i = 1, 2$ , in the interval  $(0, \infty)$

9. The value of  $2m_1 + 3n_1 + m_1n_1$  is \_\_\_\_\_.

Ans: 57

10. The value of  $6m_2 + 4n_2 + 8m_2n_2$  is \_\_\_\_\_.

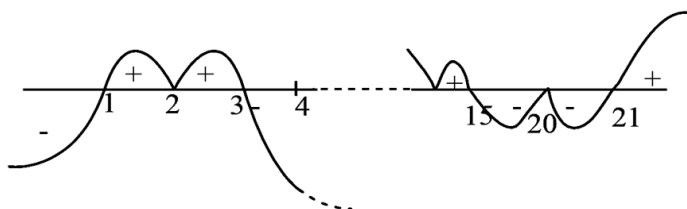
Ans: 6

**Sol (9&10Q):**

$$f_1(x) = \int_0^x (t-1)^1 (t-2)^2 (t-3)^3 (t-4)^4 \dots (t-21)^{21} dt$$

$$f_1'(x) = (x-1)(x-2)^2(x-3)^3(x-4)^4 \dots (x-21)^{21}$$

Plotting wavycurve of  $f_1'(x)$



So for  $x = 4k + 1, (k \in \mathbb{W}) f_1'(x)$  changes sign from -ve to +ve

for  $x = 4k + 3, (k \in \mathbb{W}) f_1'(x)$  changes sign from +ve to -ve

So  $m_1 = \text{no. of local minima} = 6$

$n_1 = \text{no. of local maxima} = 5$

# MOMENTUM

$$(Q9) 2m_1 + 3n_1 + m_1n_1 = 12 + 15 + 30 = 57$$

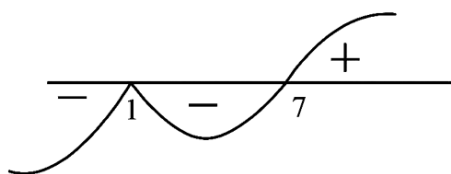
$$f_2(x) = 98(x-1)^{50} - 600(x-1)^{49} + 2450$$

$$f_2'(x) = 98 \times 50(x-1)^{49} - 600 \times 49(x-1)^{48}$$

$$= 98 \times 50(x-1)^{48}(x-1-6)$$

$$= 98 \times 50(x-1)^{48}(x-7)$$

Wavy curve of  $f_2'(x)$  is



Clearly  $m_2 = 1, n_2 = 0$

$$(10) 6m_2 + 4n_2 + 8m_2n_2 = 6 + 0 + 0 = 6$$

## Question Stem for Question Nos. 11 and 12

Let  $g_i : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \rightarrow \mathbb{R}, i=1,2$ , and  $f : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \rightarrow \mathbb{R}$  be functions such that

$$g_1(x) = 1, g_2(x) = |4x - \pi| \text{ and } f(x) = \sin^2 x, \text{ for all } x \in \left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$$

Define

$$S_i = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x) \cdot g_i(x) dx, i=1,2$$

11. The value of  $\frac{16S_1}{\pi}$  is \_\_\_\_\_.

Ans: 2

12. The value of  $\frac{48S_2}{\pi^2}$  is \_\_\_\_\_.

Ans: 1.5

**Sol(11&12Q):**

$$\begin{aligned} S_1 &= \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \sin^2 x \cdot 1 \cdot dx = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1 - \cos 2x}{2} dx \\ &= \left[ \frac{x}{2} - \frac{\sin 2x}{4} \right]_{\frac{\pi}{8}}^{\frac{3\pi}{8}} = \frac{\pi}{8} - \frac{1}{4} \left( \sin \frac{3\pi}{4} - \sin \frac{\pi}{4} \right) = \frac{\pi}{8} \end{aligned}$$

# MOMENTUM

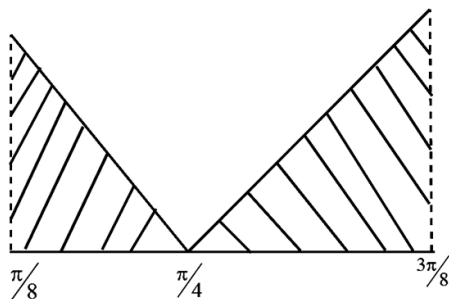
$$(Q11) \frac{16S_1}{\pi} = 2$$

$$S_2 = \int_{\pi/8}^{3\pi/8} |4x - \pi| \cdot \sin^2 x \, dx$$

Using King's Property

$$S_2 = \int_{\pi/8}^{3\pi/8} |\pi - 4x| \cdot \cos^2 x \, dx$$

$$\text{Adding } 2S_2 = \int_{\pi/8}^{3\pi/8} |\pi - 4x| \, dx$$



$$\Rightarrow 2S_2 = \text{area under graph} = 2 \times \frac{1}{2} \times \frac{\pi}{8} \times \frac{\pi}{2} = \frac{\pi^2}{16}$$

$$(Q12) \Rightarrow \frac{48S_2}{\pi^2} = \frac{24}{16} = 1.5$$

## SECTION-3(Maximum Marks: 12)

### Paragraph with Single Answer Type

- This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks : +3 If ONLY the correct option is chosen;*

*Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);*

*Negative Marks : -1 In all other cases.*

### **Paragraph-1:**

$$\text{Let } M = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 \leq r^2\},$$

where  $r > 0$ . Consider the geometric progression  $a_n = \frac{1}{2^{n-1}}, n = 1, 2, 3, \dots$ . Let  $S_0 = 0$

and, for  $n \geq 1$ , let  $S_n$  denote the sum of the first  $n$  terms of this progression. For  $n \geq 1$ ,

# MOMENTUM

let  $C_n$  denote the circle with center  $(S_{n-1}, 0)$  and radius  $a_n$ , and  $D_n$  denote the circle with center  $(S_{n-1}, S_{n-1})$  and radius  $a_n$ .

13. Consider  $M$  with  $r = \frac{1025}{513}$ . Let  $k$  be the number of all those circles  $C_n$  that are inside  $M$ . Let  $l$  be the maximum possible number of circles among these  $k$  circles such that no two circles intersect. Then  
 A)  $k + 2l = 22$       B)  $2k + l = 26$       C)  $2k + 3l = 34$       D)  $3k + 2l = 40$

Ans: D

14. Consider  $M$  with  $r = \frac{(2^{199} - 1)\sqrt{2}}{2^{198}}$ . The number of all those circles  $D_n$  that are inside  $M$  is  
 A) 198                      B) 199                      C) 200                      D) 201

Ans: B

**Sol(13&14Q):**

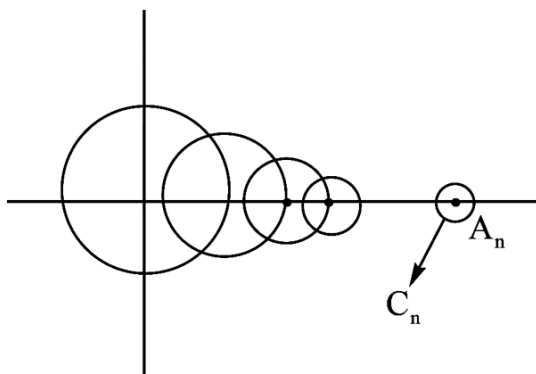
$$S_n = \sum_{n=1}^n a_n = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \dots + \frac{1}{2^{n-1}}$$

$$= \frac{1 - \frac{1}{2^n}}{1 - \frac{1}{2}} = \frac{2^n - 1}{2^{n-1}}$$

For  $C_n$ , centre of circle is  $\left(\frac{2^{n-1} - 1}{2^{n-2}}, 0\right)$

Radius of circle  $= \frac{1}{2^{n-1}}$

Plotting circles,



Finding bigger x intercept of  $C_n$

$$= \frac{2^{n-1} - 1}{2^{n-2}} + \frac{1}{2^{n-1}} = \frac{2^n - 2 + 1}{2^{n-1}} = \frac{2^n - 1}{2^{n-1}}$$

# MOMENTUM

For  $C_n$  to be inside  $M$  for  $r = \frac{1025}{513} = \frac{2^{10} + 1}{2^9 + 1}$

$$\frac{2^n - 1}{2^{n-1}} \leq \frac{2^{10} + 1}{2^9 + 1}$$

$$\Rightarrow 2 - \frac{1}{2^{n-1}} \leq \frac{2^{10} + 1}{2^9 + 1} \quad \Rightarrow \frac{1}{2^{n-1}} \geq \frac{1}{2^9 + 1}$$

$$\Rightarrow 2^{n-1} \leq 2^9 + 1 \Rightarrow n - 1 \leq 9 \Rightarrow n \leq 10$$

Hence no. Of circles possible = 10

For non intersecting pair of circles we need to choose alternating circles. Hence maximum 5 circles can be chosen.

So.  $k = 10, l = 5$

$$3k + 2l = 40$$

For  $D_n$  centre is  $\left( \frac{2^{n-1} - 1}{2^{n-2}}, \frac{2^{n-1} - 1}{2^{n-2}} \right)$

$$\text{Radius} = \frac{1}{2^{n-1}}$$

Max distance of a point on  $D_n$  from origin.

= (Distance of centre from origin) + (radius)

$$= \frac{2^{n-1} - 1}{2^{n-2}} \sqrt{2} + \frac{1}{2^{n-1}} = \frac{(2^n - 2)\sqrt{2} + 1}{2^{n-1}}$$

So for  $D_n$  to be inside  $M$  with  $r = \left( \frac{2^{199} - 1}{2^{198}} \right) \sqrt{2}$

$$\frac{(2^n - 2)\sqrt{2} + 1}{2^{n-1}} \leq \frac{(2^{199} - 1)\sqrt{2}}{2^{198}}$$

$$\Rightarrow \frac{(2^{n-1} - 1)\sqrt{2}}{2^{n-2}} \frac{(2^n - 2)\sqrt{2} + 1}{2^{n-1}} \leq \frac{(2^{199} - 1)\sqrt{2}}{2^{198}} \quad \Rightarrow n < 200$$

Hence 199 circles are possible

## Paragraph-II:

Let  $\Psi_1 : [0, \infty) \rightarrow \mathbb{R}, \Psi_2 : [0, \infty) \rightarrow \mathbb{R}, f : [0, \infty) \rightarrow \mathbb{R}$  and  $g : [0, \infty) \rightarrow \mathbb{R}$  be functions such that  $f(0) = g(0) = 0$ ,

$$\Psi_1(x) = e^{-x} + x, x \geq 0,$$

$$\Psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \geq 0,$$

$$f(x) = \int_{-x}^x (|t| - t^2) e^{-t^2} dt, x > 0$$

and

# MOMENTUM

$$g(x) = \int_0^{x^2} \sqrt{t} e^{-t} dt, x > 0.$$

15. Which of the following statements is TRUE?

A)  $f(\sqrt{\ln 3}) + g(\sqrt{\ln 3}) = \frac{1}{3}$

B) For every  $x > 1$ , there exists an  $\alpha \in (1, x)$  such that  $\Psi_1(x) = 1 + \alpha x$

C) For every  $x > 0$ , there exists a  $\beta \in (0, x)$  such that  $\Psi_2(x) = 2x(\Psi_1(\beta) - 1)$

D)  $f$  is an increasing function on the interval  $\left[0, \frac{3}{2}\right]$

Ans: C

Sol:

$$\Psi_1(x) = e^{-x} + x, x \geq 0, \quad \Psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \geq 0$$

$$f(x) = \int_{-x}^x (|t| - t^2) e^{-t^2} dt, x > 0$$

$$\Rightarrow f(x) = 2 \int_0^x (|t| - t^2) e^{-t^2} dt, x > 0 \quad = 2 \int_0^x (t - t^2) e^{-t^2} dt \dots\dots(1)$$

$$g(x) = \int_0^{x^2} \sqrt{t} \cdot e^{-t} dt, x > 0$$

$$t = z^2 \quad = \int_0^x z \cdot e^{-z^2} \cdot 2z \cdot dz$$

$$= 2 \int_0^x z^2 \cdot e^{-z^2} \cdot dz \quad = 2 \int_0^x t^2 \cdot e^{-t^2} \cdot dt \quad \dots\dots(2)$$

$$f'(x) = 2(x - x^2) e^{-x^2}$$

$$= 2x(1 - x) e^{-x^2}$$

$$f \uparrow \text{ for } x \in (0, 1)$$

$$f \downarrow \text{ for } x \in (1, \infty) \quad (\text{Option D is wrong})$$

$$\therefore f(x) + g(x) = 2 \int_0^x t \cdot e^{-t^2} dt$$

$$= \left[ -e^{-t^2} \right]_0^x = (-e^{-x^2}) - (-1) = 1 - e^{-x^2}$$

$$\therefore f(x) + g(x) = 1 - e^{-x^2}$$

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$$\therefore f(\sqrt{\ln 3}) + g(\sqrt{\ln 3}) = 1 - \frac{1}{3} = \frac{2}{3}$$

(Option A wrong)

$$\Psi_1(x) = e^{-x} + x, x \geq 0$$

$$\Psi_1'(x) = 1 - e^{-x} \geq 0 \Rightarrow \Psi_1(x) \text{ is increasing}$$

$$\Psi(0) = 1$$

$$\Psi_1'(0) = 0$$

$$\Psi_1''(x) = e^{-x} > 0$$

$\therefore \Psi_1(x)$  is concave up

**Method-I:**

$$\Psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \geq 0$$

$$\Psi_2(0) = 0$$

$$\Psi_2'(x) = 2x - 2 + 2e^{-x}$$

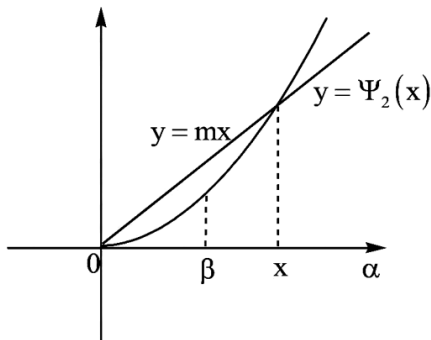
$$= 2(x - 1 + e^{-x})$$

$$\Psi_2''(x) = 2(1 - e^{-x}) > 0 \text{ (concave up)}$$

$$\Psi_2(x) = 2(\Psi_1(\beta) - 1)x$$

$$\Psi_2(x) = mx \text{ (} m > 0 \text{)}$$

Option C correct



**Method-2:**

LMVT

$$\Psi_2'(\beta) = \frac{\Psi_2(x) - \Psi_2(0)}{x - 0} \Rightarrow 2(\Psi_1(\beta) - 1) = \frac{\Psi_2(x) - 0}{x - 0} \Rightarrow \Psi_2(x) = 2(\Psi_1(\beta) - 1)x$$

16. Which of the following statements is TRUE?

A)  $\Psi_1(x) \leq 1$ , for all  $x > 0$

B)  $\Psi_2(x) \leq 0$ , for all  $x > 0$

C)  $f(x) \geq 1 - e^{-x^2} - \frac{2}{3}x^3 + \frac{2}{5}x^5$ , for all  $x \in \left(0, \frac{1}{2}\right)$

# MOMENTUM

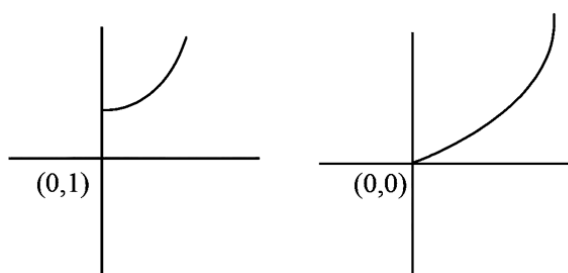
D)  $g(x) \leq \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{1}{7}x^7$ , for all  $x \in \left(0, \frac{1}{2}\right)$

Ans: D

Sol:  $g(x) = 2 \int_0^x t^2 \left(1 - t^2 + \frac{t^4}{2!} \dots\right) dt$

$$= 2 \int_0^x \left(t^2 - t^4 + \frac{t^6}{2!} \dots\right) dt = 2 \left[ \frac{t^3}{3} - \frac{t^5}{5} + \frac{t^7}{7 \cdot 2!} \dots \right]_0^x$$

$$= 2 \left[ \frac{x^3}{3} - \frac{x^5}{5} + \frac{x^7}{7 \cdot 2!} \dots \right] = \frac{2x^3}{3} - \frac{2}{5}x^5 + \frac{x^7}{7} \dots \leq \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{x^7}{7}$$



$\Psi_1(x) = e^{-x} + x, x \geq 0$

$\Psi_1(x) \geq 1$

Option A wrong  $\Psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \geq 0$

$\Psi_2(x) \geq 0$   $\therefore$  Option B wrong also Option C wrong

## SECTION-4(Maximum Marks: 12)

### Non-Negative Integer Answer Type

- This section contains THREE (03) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If ONLY the correct integer is entered;

Zero Marks : 0 In all other cases.

17. A number is chosen at random from the set  $\{1, 2, 3, \dots, 2000\}$ . Let  $p$  be the probability that the chosen number is a multiple of 3 or a multiple of 7. Then the value of  $500p$  is \_\_\_\_\_.

Ans: 214

Sol:  $\{1, 2, 3, \dots, 2000\}$

$E_1 =$  Event that it is a multiple of 3

$E_2 =$  Event that it is a multiple of 7

$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

# MOMENTUM

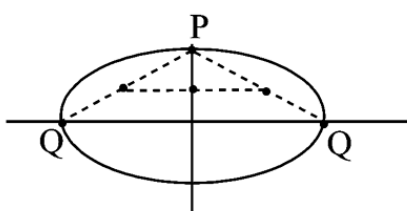
$$= \frac{666 + 285 - 95}{2000} = \frac{856}{2000}$$

$$\therefore GE = 500 \times \frac{856}{2000} = \frac{856}{4} = 214$$

18. Let  $E$  be the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ . For any three distinct points  $P, Q$  and  $Q'$  on  $E$ , let  $M(P, Q)$  be the mid-point of the line segment joining  $P$  and  $Q$ , and  $M(P, Q')$  be the mid-point of the line segment joining  $P$  and  $Q'$ . Then the maximum possible value of the distance between  $M(P, Q)$  and  $M(P, Q')$ , as  $P, Q$  and  $Q'$  vary on  $E$ , is \_\_\_\_\_

Ans: 4

Sol: Maximum chord  $= 2a = 8$



Required distance between

$$M(P, Q), M(P, Q') = \frac{1}{2}(8) = 4$$

19. For any real number  $x$ , let  $[x]$  denote the largest integer less than or equal to  $x$ . If

$$I = \int_0^{10} \left[ \sqrt{\frac{10x}{x+1}} \right] dx, \text{ then the value of } 9I \text{ is } \underline{\hspace{2cm}}.$$

Ans: 182

Sol:  $\phi(x) = \frac{10x}{x+1} \Rightarrow \phi'(x) > 0 \Rightarrow \phi \uparrow \quad \therefore \phi(0) = 0, \phi(10) = \frac{100}{11} = 9.01$

$$\sqrt{\frac{10x}{x+1}} \in [0, 3.01] \quad \frac{10x}{x+1} = 1 \text{ (or) } 4 \text{ (or) } 9$$

$$9x = 1 \quad 10x = 4x + 4 \quad 10x = 9x + 9$$

$$x = \frac{1}{9} \quad x = \frac{2}{3} \quad x = 9$$

$$GI = \int_0^{1/9} 0 \cdot dx + \int_{1/9}^{2/3} 1 \cdot dx + \int_{2/3}^9 2 \cdot dx + \int_9^{10} 3 \cdot dx = 0 + \left( \frac{2}{3} - \frac{1}{9} \right) + 2 \left( 9 - \frac{2}{3} \right) + 3(10 - 9)$$

$$= \frac{5}{9} + \frac{50}{3} + 3 = \frac{5 + 150 + 27}{9} = \frac{182}{9} \Rightarrow 9I = 182$$