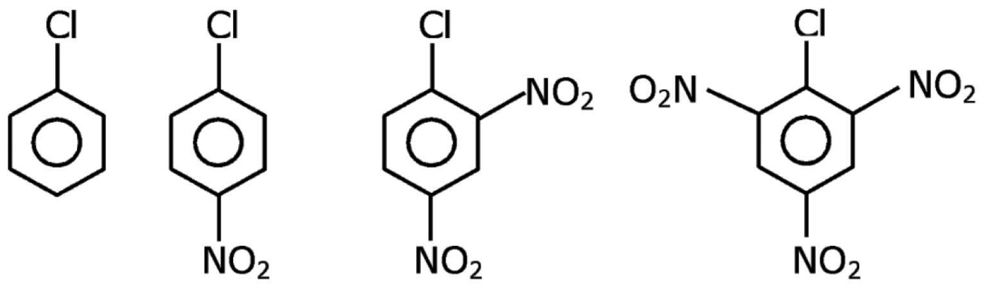
MOMENTUM

1. The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is:

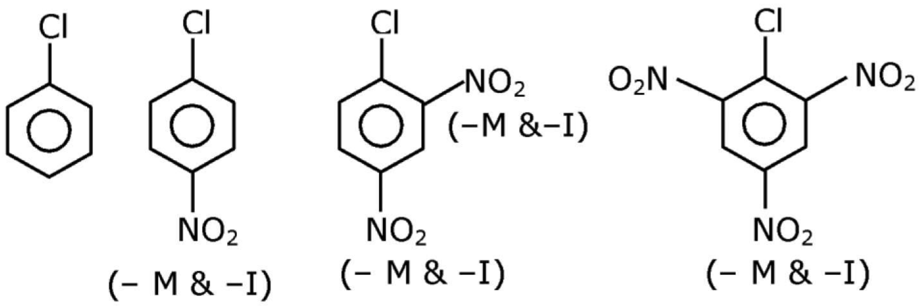


(i) (ii) (iii) (iv)

a. (iv) < (i) < (iii) < (ii)
b. (iv) < (i) < (ii) < (iii)
c. (i) < (ii) < (iii) < (iv)
d. (iv) < (iii) < (ii) < (i)

Ans (c)

Solution:



(i) (ii) (iii) (iv)

Reactivity ∝ – M group present at o/p position.

2. Match List-I with List-II

List- I (Metal)	List-II (Ores)
(a) Aluminum	(i) Siderite
(b) Iron	(ii) Calamine
(c) Copper	(iii) Kaolinite
(d) Zinc	(iv) Malachite

a. (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
b. (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
c. (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
d. (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Ans (c)
Solution:

Siderite	$FeCO_3$
Calamine	$ZnCO_3$
Kaolinite	$Si_2Al_2O_5(OH)_4$ or $Al_2O_3.2SiO_2.2H_2O$
Malachite	$CuCO_3.Cu(OH)_2$

3. Match List-I with List-II

List- I (Salt)	List-II (Flame colour wavelength)
(a) LiCl	(i) 455.5 nm
(b) NaCl	(ii) 970.8 nm
(c) RbCl	(iii) 780.0 nm
(d) CsCl	(iv) 589.2 nm

Choose the correct answer from the options given below:
a. (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
b. (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
c. (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)
d. (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

Ans (b)
Solution:
Range of visible region: -
390 nm – 760 nm

VIBGYOR

Violet - Red

LiCl	Crimson Red
NaCl	Golden yellow
RbCl	Violet
CsCl	Blue

So, LiCl which is crimson have wave length closed to red in the spectrum of visible region which is as per given data.

4. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Hydrogen is the most abundant element in the Universe, but it is not the most abundant gas in the troposphere.

Reason R: Hydrogen is the lightest element.

In the light of the above statements, choose the correct answer from the given below

(1) A is false but R is true
(2) Both A and R are true and R is the correct explanation of A
(3) A is true but R is false
(4) Both A and R are true but R is NOT the correct explanation of A

a. A is false but R is true
b. Both A and R are true and R is the correct explanation of A
c. A is true but R is false
d. Both A and R are true but R is NOT the correct explanation of A

Ans (b)

Solution:

Hydrogen is most abundant element in universe because all luminous body of universe i.e. stars & nebulae are made up of hydrogen which acts as nuclear fuel & fusion reaction is responsible for their light.

5. Given below are two statements:

Statement I: The value of the parameter "Biochemical Oxygen Demand (BOD)" is important for survival of aquatic life.

Statement II: The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the most appropriate answer from the options given below.

a. Both Statement I and Statement II are false
b. Statement I is false but Statement II is true
c. Statement I is true but Statement II is false
d. Both Statement I and Statement II are true

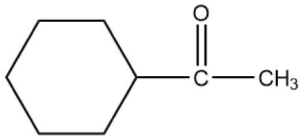
Ans (c)

Solution:

For survival of aquatic life dissolved oxygen is responsible its optimum limit 6.5 ppm and optimum limit of BOD ranges from 10-20 ppm & BOD stands for biochemical oxygen demand.

6. Which one of the following carbonyl compounds cannot be prepared by addition of water on an alkyne in the presence of $HgSO_4$ and H_2SO_4?

a. $CH_3-CH_2-\overset{O}{\overset{||}{C}}-H$

b. 

c. $CH_3-\overset{O}{\overset{||}{C}}-H$

d. $CH_3-\overset{O}{\overset{||}{C}}-CH_2CH_3$

Ans (a)

Solution:

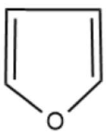
Reaction of Alkyne with $HgSO_4$ & H_2SO_4 follow as

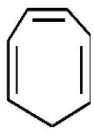
$$CH\equiv CH \xrightarrow[H_2O]{HgSO_4, H_2SO_4} CH_3CHO$$

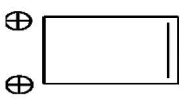
$$CH_3-C\equiv CH \xrightarrow[H_2O]{HgSO_4, H_2SO_4} CH_3-\overset{O}{\overset{||}{C}}-CH_3$$

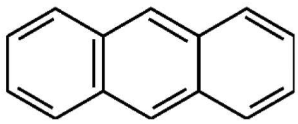
Hence, by this process preparation of CH_3CH_2CHO can't be possible.

7. Which one of the following compounds is non-aromatic?

a. 

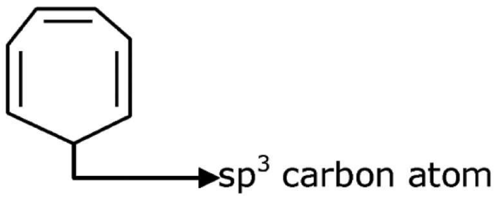
b. 

c. 

d. 

Ans (b)

Solution:



(Not planar)

Hence, it is non-aromatic.

8. The incorrect statement among the following is:
   a. $VOSO_4$ is a reducing agent
   b. Red color of ruby is due to the presence of CO^{3+}
   c. Cr_2O_3 is an amphoteric oxide
   d. RuO_4 is an oxidizing agent

Ans (b)
Solution:

Red color of ruby is due to presence of CrO_3 or Cr^{+6} not CO^{3+}

9. According to Bohr's atomic theory:
   (a) Kinetic energy of electron is $\propto \frac{Z^2}{n^2}$
   (b) The product of velocity (v) of electron and principal quantum number (n). 'v_n' $\propto Z^2$
   (c) Frequency of revolution of electron in an orbit is $\propto \frac{Z^3}{n^3}$
   (d) Coulombic force of attraction on the electron is $\propto \frac{Z^3}{n^4}$

   Choose the most appropriate answer from the options given below:
   a. (c) only
   b. (a) and (d) only
   c. (a) only
   d. (a), (c) and (d) only

Ans (b)

Solution:

(a) $KE = -TE = 13.6 \times \frac{Z^2}{n^2}\ eV$

$KE \propto \frac{Z^2}{n^2}$

(b) $V = 2.188 \times 10^6 \times \frac{Z}{n}\ m/s$

So, $V_n \propto Z$

Frequency $= \frac{V}{2\pi r}$

$$F \propto \frac{Z^2}{n^3}\left[\because r \propto \frac{n^2}{z} \text{ and } v \propto \frac{Z}{n}\right]$$

(d) Force $\propto \frac{Z^2}{r^2}$

So, $F \propto \frac{Z^3}{n^4}$

So, only statement (A) is correct.

10. Match List-I with List-II

List- I	List-II
(a) Valium	(i) Antifertility drug
(b) Morphine	(ii) Pernicious anaemia
(c) Norethindrone	(iii) Analgesic
(d) Vitamin B12	(iv) Tranquilizer

a. (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
b. (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)
c. (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
d. (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

Ans (d)
Solution:

(a) Valium	(iv) Tranquilizer
(b) Morphine	(iii) Analgesic
(c) Norethindrone	(i) Antifertility drug
(d) Vitamin B12	(ii) Pernicious anemia

11. The Correct set from the following in which both pairs are in correct order of melting point is
a. LiF > LiCl ; NaCl > MgO
b. LiF > LiCl ; MgO > NaCl
c. LiCl > LiF ; NaCl > MgO
d. LiCl > LiF ; MgO > NaCl

Ans (b)

Solution:
Generally

M.P. $\propto$ Lattice energy = $\frac{KQ_1Q_2}{r^+ + r^-}$

$\propto$ (packing efficiency)

12. The calculated magnetic moments (spin only value) for species $[FeCl_4]^{2-}$, $[Co(C_2O_4)_3]^{3-}$ and MnO_4^{2-} respectively are:

a. 5.92, 4.90 and 0 BM
b. 5.82, 0 and 0 BM
c. 4.90, 0 and 1.73 BM
d. 4.90, 0 and 2.83 BM

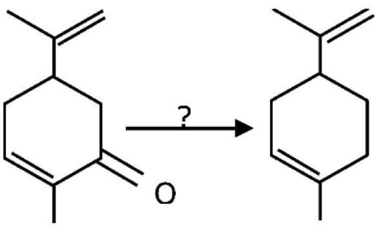
Ans (c)
Solution:
$[FeCl_4]^{2-}$

Fe^{2+} $3d^6$ → 4 unpaired electrons. as Cl^- in a weak field liquid.

$\mu_{spin} = \sqrt{24}$ BM

= 4.9 BM

13.



Which of the following reagent is suitable for the preparation of the product in the above reaction?

a. Red P + Cl_2

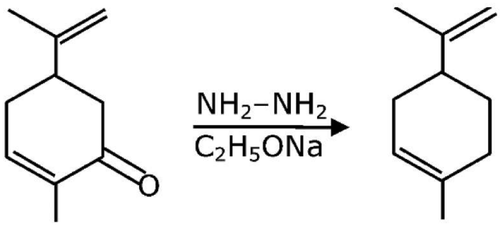
b. NH_2-NH_2/ $C_2H_5O^-Na^+$

c. Ni/H_2

d. $NaBH_4$

Ans: (b)

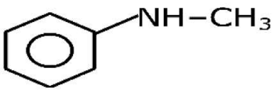
Solution:



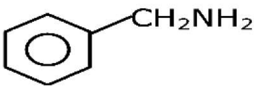
It is wolff-kishner reduction of carbonyl compounds.

14. The diazonium salt of which of the following compounds will form a coloured dye on reaction with β-Naphthol in NaOH?

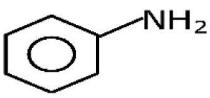
a.



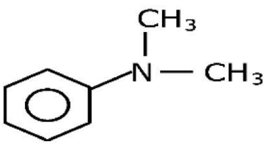
b.



c.

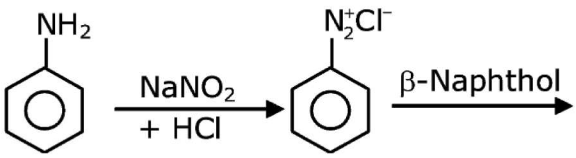


d.



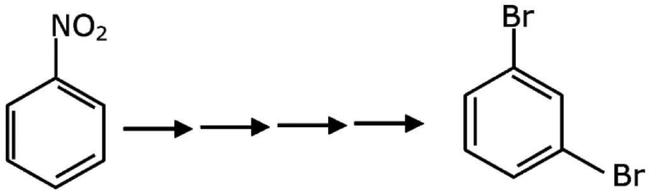
Ans: (c)

Solution:



Orange bright dye.

15. What is the correct sequence of reagents used for converting nitrobenzene into m-dibromobenzene?



a. $\xrightarrow{Sn/HCl}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{NaNO_2}$ / $\xrightarrow{NaBr}$

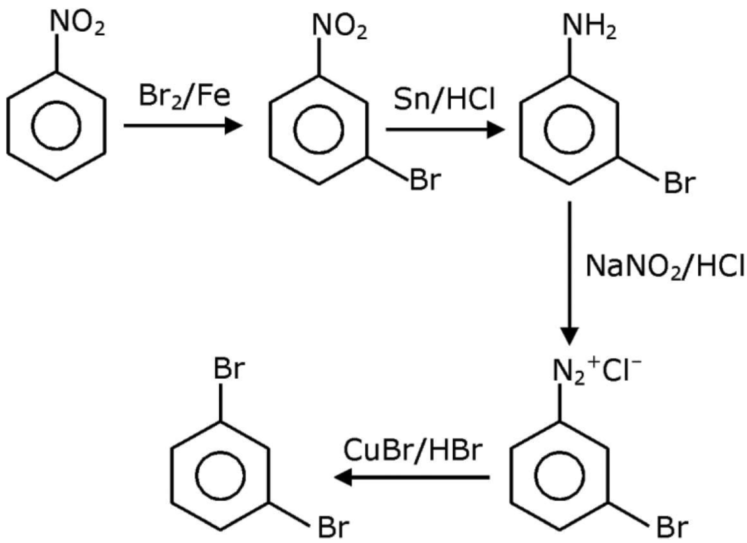
b. $\xrightarrow{Sn/HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{H^+}$

c. $\xrightarrow{NaNO_2}$ / $\xrightarrow{HCl}$ / $\overline{\text{KE}}$

d. $\xrightarrow{Br_2/Fe}$ / $\xrightarrow{Sn/HCl}$ / $\xrightarrow{NaNO_2/HCl}$ / $\xrightarrow{CuBr/HBr}$

Ans: (d)

Solution:

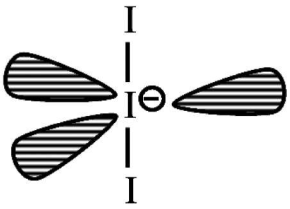


16. The correct shape and I-I-I bond angles respectively in I_3^- ion are:
   a. Trigonal planar; 120°
   b. Distorted trigonal planar; 135° and 90°
   c. Linear; 180º
   d. T-shaped; 180º and 90º

Ans: (c)

Solution:

I_3^- has sp^{3}d hybridization (2 BP + 3 LP) and linear geometry.



17. What is the correct order of the following elements with respect to their density?

a. Cr < Fe < Co < Cu < Zn   b. Cr < Zn < Co < Cu < Fe

c. Zn < Cu < Co < Fe < Cr   d. Zn < Cr < Fe < Co < Cu

Ans: (d)

Solution:

Fact Based

Density depends on many factors like atomic mass. atomic radius and packing efficiency.

18. Match List-I and List-II.

List – I	List-II
a. $R-\overset{\overset{\large O}{\|\|}}{C}-Cl \rightarrow R-CHO$	(i) Br_2/NaOH
b. $R-CH_2-COOH \rightarrow R-\underset{\underset{\large Cl}{\|}}{CH}-COOH$	(ii) H_2/Pd-$BaSO_4$
c. $R-\overset{\overset{\large O}{\|\|}}{C}-CH_3 \rightarrow R-CH_2-CH_3$	(iii) Zn (Hg)/Conc. HCl
d. $R-\overset{\overset{\large O}{\|\|}}{C}-NH_2 \rightarrow R-NH_2$	(iv) Cl_2/Red P, H_2O

a. (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
b. (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
c. (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
d. (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Ans: (d)
Solution:

(a) $R-\overset{\overset{O}{||}}{C}-Cl \xrightarrow{H_2/Pd-BaSO_4} R-CHO$ (Rosenmund reaction)

(b)

$R-CH_2-COOH \xrightarrow{Cl_2/Red\ P,\ H_2O} R-\underset{\underset{Cl}{|}}{CH}-COOH$ (HVZ reaction)

(c)

$R-\overset{\overset{O}{||}}{C}-NH_2 \xrightarrow{Br_2/NaOH} R-NH_2$ (Hoffmann Bromamide reaction)

(d)

$R-\overset{\overset{O}{||}}{C}-CH_3 \xrightarrow{Zn(Hg)/conc.HCl} R-CH_2-CH_3$ (Clemmensen reaction)

19. In polymer Buna-S: 'S' stands for :
a. Styrene
b. Sulphur
c. Strength
d. Sulphonation

Ans: (a)
Solution:
Buna-S is the co-polymer of buta-1,3-diene & styrene

20. Most suitable salt which can be used for efficient clotting of blood will be:
   a. $Mg(HCO_3)_2$ b. $FeSO_4$
   c. $NaHCO_3$ d. $FeCl_3$

Ans: (d)

Solution:
Blood is a negative sol, according to Hardy-Schulz's rule, the cation with high charge has high coagulation power. Hence, $FeCl_3$ can be used for clotting blood.

Section B

1. The magnitude of the change in oxidising power of the MnO_4^-/Mn^{2+} couple is $x \times 10^{-4}$ V, if the H^+ concentration is decreased from 1M to 10^{-4} M at 25°C. (Assume concentration of MnO_4^- and Mn^{2+} to be same on change in H^+ concentration). The value of x is _____.
(Rounded off to the nearest integer)
$\left[\text{Given : } \frac{2303RT}{F} = 0.059\right]$

Ans: 3776
Solution:

$$5e^- + MnO_4^- + 8H^+ \longrightarrow Mn^{+2} + 4H_2O$$

$$Q = \frac{[Mn^{+2}]}{[H^+]^8[MnO_4^-]} \quad E_1 = E^\circ - \frac{0.059}{5}\log(Q_1)$$

$$E_2 = E^\circ - \frac{0.059}{5}\log(Q_2) \quad E_2 - E_1 = \frac{0.059}{5}\log\left(\frac{Q_1}{Q_2}\right)$$

$$\frac{0.059}{5}\log\left\{\frac{[H^+]_{II}}{[H^+]_I}\right\}^8 \quad \frac{0.059}{5}\log\left(\frac{10^{-4}}{1}\right)^8$$

$$(E_2 - E_1) = \frac{0.059}{5}\times(-32) \quad |(E_2 - E_1)| = 32\times\frac{0.059}{5} = x\times10^{-4}$$

$$\frac{32\times590}{5}\times10^{-4} = x\times10^{-4}$$

$= 3776 \times 10^{-4}$ so, $x = 3776$

2. Among the following allotropic forms of sulphur, the number of allotropic forms, which will show paramagnetism is ______.
   (1) α-sulphur (2) β-sulphur (3) S_2-form

Ans: 1
Solution:
S_2 is like O_2 i.e. paramagnetic as per molecular orbital theory.

3. C_6H_6 freezes at 5.5ºC. The temperature at which a solution of 10 g of C_4H_{10} in 200 g of C_6H_6 freeze is ________ °C. (The molal freezing point depression constant of C_6H_6 is 5.12°C/m).

Ans: 1
Solution:
$\Delta T_f = i \times K_f \times m$
$$= 1 \times 5.12 \times \frac{10/58}{200} \times 1000$$
$$\Delta T_f = \frac{5.12 \times 50}{58} = 4.414$$
$T_{f(solution)} = T_{K(solvent)} - \Delta T_f = 5.5 - 4.414 = 1.086°C$
$\approx 1.09°C = 1$ (nearest integer)

4. The volume occupied by 4.75 g of acetylene gas at 50°C and 740 mmHg pressure is _______L. (Rounded off to the nearest integer)
   (Given R = 0.0826 L atm K^{-1} mol^{-1})

Ans: 5
Solution:
T = 50°C = 323.15 K
$P = 740 \text{ mm of Hg} = \frac{740}{760}\ atm$
V = ?
$\text{moles (n)} = \frac{4.75}{26}\ atm$
$$V = \frac{4.75}{26} \times \frac{0.0821 \times 323.15}{740} \times 760$$
$V = 4.97 \approx 5$ Lit

5. The solubility product of PbI_2 is 8.0×10^{-9}. The solubility of lead iodide in 0.1 molar solution of lead nitrate is $x \times 10^{-6}$ mol/L. The value of x is _______ (Rounded off to the nearest integer)
Given $\sqrt{2} = 1.41$

Ans: 141

Solution: $PbI_2(s) \rightleftharpoons Pb^{2+}(aq) + 2I^-(aq)$

S+0.1 2s

$K_{SP}(PbI_2) = 8 \times 10^{-9}$

$K_{SP} = [Pb^{+2}][I^-]^2$

$8 \times 10^{-9} = (S + 0.1)(2S)^2 \Rightarrow (8 \times 10^{-9} + 0.1) \times 4S^2$

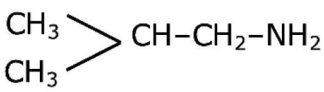
$\Rightarrow S^2 = 2 \times 10^{-8}$

$S = 1.414 \times 10^{-4}$ mol/Lit

$= x \times 10^{-6}$ mol/Lit $\therefore$ $x = 141.4 \approx 141$

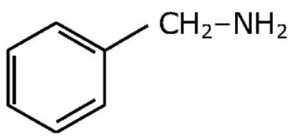
6. The total number of amines among the following which can be synthesized by Gabriel synthesis is ______.

1.

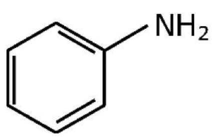


2. $CH_3CH_2NH_2$

3.



4.



Ans: 3
Solution:

Only 1° amines can be prepared by Gabriel synthesis.

7. 1.86 g of aniline completely reacts to form acetanilide. 10% of the product is lost during purification. Amount of acetanilide obtained after purification (in g) is
 ____× 10–2.

Ans: 243
Solution:

$$\underset{\substack{(C_6H_7N) \\ \text{Molar mass} = 93}}{Ph-NH_2} \longrightarrow \underset{\substack{(\text{Acetanilide})(C_8H_9NO) \\ \text{Molar mass} = 135}}{Ph-NH-\overset{\overset{\displaystyle O}{||}}{C}-CH_3}$$

93 g Aniline produce 135 g acetanilide

1.86 g produce $\frac{135\times1.86}{93} = 2.70\ g$

At 10% loss, 90% product will be formed after purification.

$\therefore$ Amount of product obtained $= \frac{2.70\times90}{100} = 2.43\ g\ = 243 \times 10^{-2}$ g

8. The formula of a gaseous hydrocarbon which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 times its own volume of CO_2 is C_xH_y. The value of y is ………..

Ans: 8
Solution:

$C_xH_y + 6O_2 \rightarrow 4CO_2 + \frac{y}{2} H_2O$

Applying POAC on 'O' atoms

$6 \times 2 = 4 \times 2 + y/2 \times 1$

$\frac{y}{2} = 4 \Rightarrow y = 8$

9. Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25ºC. After 9h, the fraction of sucrose remaining is f. The value of $\log_{10}\frac{1}{2}$ is ________ $\times 10^{-2}$ (Rounded off to the nearest integer)

[Assume: ln10 = 2.303, ln2 = 0.693]

Ans : 81

Solution:

Sucrose $\xrightarrow{\text{Hydrolysis}}$ Glucose + Fructose

$t_{1/2} = 3.33\text{h} = \frac{10}{3}h$

$C_t = \frac{C_o}{2^{t/t_{1/2}}}$

Fraction of sucrose remaining = f $= \frac{C_t}{C_o} = \frac{1}{2^{\frac{t}{t\frac{1}{2}}}}$

$\frac{1}{f} = 2^{t/t_{1/2}}$

$\log(1/f) = \log\left(2^{t/t_{1/2}}\right) = \frac{t}{t_{1/2}}\log(2)$

$\frac{9}{10/3} \times 0.3 = \frac{8.1}{10} = 0.81$

$= x \times 10^{-2}$ $\quad x = 81$

10. Assuming ideal behavior, the magnitude of log K for the following reaction at 25ºC is x × 10^{-1}. The value of x is _________.(Integer answer)

$3HC \equiv CH(g) \rightleftharpoons C_6H_6\ (l)$

[Given: $\Delta_f G°(HC \equiv CH) = -2.04 \times 10^5$] mol^{-1}; $\Delta_f G°(C_6H_6) = -1.24 \times 10^5\ J\ mol^{-1}$;

$R = 8.314\ J\ K^{-1}\ mol^{-1}$]

Ans: 855

Solution:

$\Delta G_r^o = \Delta G_f^o\left[C_6H_8(\ell)\right] - 3 \times \Delta G_f^o\left[HC \equiv CH\right]$

$= [-1.24 \times 10^5 - 3x\ (-2.04 \times 10^5)]$

$= 4.88 \times 10^5$ J/mol

$\Delta G_r^o = -RT \ln (K_{eq})$

$\log (K_{eq}) = \frac{-\Delta G^O}{2.303RT}$

$\frac{-4.88 \times 10^5}{2.303 \times 8.314 \times 298} = -8.55 \times 101 = 855 \times 10^{-1}$

1. Let $a, b \in \mathbf{R}$. If the mirror image of the point $P(a,6,9)$ with respect to the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a+b|$ is equal to :

(1) 86 (2) 88 (3)84 (4)90

Ans. **(2)**

Sol. P(a, 6, 9), Q (20, b, –a–9)

Mid point of $PQ = \left(\frac{a+20}{2}, \frac{b+6}{2}, -\frac{a}{2}\right)$ lie on the line.

$\frac{\frac{a+20}{2}-3}{7}=\frac{\frac{b+6}{2}-2}{5}=\frac{-\frac{a}{2}-1}{-9}$

$\Rightarrow \frac{a+20-6}{14}=\frac{b+6-4}{10}=\frac{-a-2}{-18}$

$\Rightarrow \frac{a+14}{14}=\frac{a+2}{18}$

$\Rightarrow 18a+252=14a+28$

$\Rightarrow 4a=-224$

$\boxed{a=-56}$

$\frac{b+2}{10}=\frac{a+2}{18}$

$\Rightarrow \frac{b+2}{10}=\frac{-54}{18}$

$\Rightarrow \frac{b+2}{10}=-3 \Rightarrow b=-32$

$|a+b|=|-56-32|=88$

2. Let f be a twice differentiable function defined on $\mathbf{R}$ such that $f(0)=1, f'(0)=2$ and $f'(x) \neq 0$ for all $x \in \mathbf{R}$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix}=0$, for all $x \in \mathbf{R}$, then the value of $f(1)$ lies in the interval :

(1) (9, 12) (2) (6, 9) (3)(3, 6) (4)(0, 3)

Ans. **(2)**

Sol. Given $f(x)f''(x)-\left(f'(x)\right)^2=0$

Let $h(x)=\frac{f(x)}{f'(x)}$

Then $h'(x)=0 \Rightarrow h(x)=k$

$\Rightarrow \frac{f(x)}{f'(x)}=k \quad \Rightarrow f(x)=kf'(x)$

$\Rightarrow f(0)=kf'(0)$

$\Rightarrow k=\frac{1}{2}$

Now, $f(x)=\frac{1}{2}f'(x)$

$\Rightarrow \int 2\,dx = \int \frac{f'(x)}{f(x)}dx$

$\Rightarrow 2x = \ln|f(x)| + C$

As $f(0) = 1 \Rightarrow C = 0$

$\Rightarrow 2x = \ln|f(x)|$

$\Rightarrow f(x) = \pm e^{2x}$

As $f(0) = 1 \Rightarrow f(x) = e^{2x}$

$\therefore f(1) = e^2 \approx 7.38$

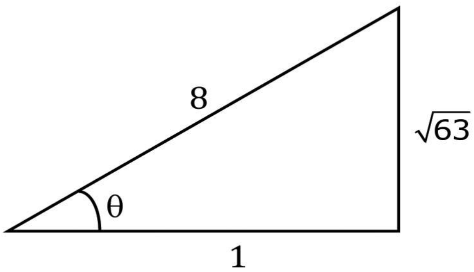
3. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is :

(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{7}}$ (3) $\sqrt{7}-1$ (4) $2\sqrt{2}-1$

Ans. (2)

Sol. $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right) = \theta$ $\sin\theta = \frac{\sqrt{63}}{8}$



$\cos\theta = \frac{1}{8}$

$2\cos^2\frac{\theta}{2} - 1 = \frac{1}{8}$

$\Rightarrow \cos^2\frac{\theta}{2} = \frac{9}{16}$

$\cos\frac{\theta}{2} = \frac{3}{4}$

$\Rightarrow \frac{1-\tan^2\frac{\theta}{4}}{1+\tan^2\frac{\theta}{4}} = \frac{3}{4}$

$\tan\frac{\theta}{4} = \frac{1}{\sqrt{7}}$

4. The probability that two randomly selected subsets of the set $\{1, 2, 3, 4, 5\}$ have exactly two elements in their intersection, is :

(1) $\frac{65}{2^7}$ (2) $\frac{135}{2^9}$ (3) $\frac{65}{2^8}$ (4) $\frac{35}{2^7}$

Ans. **(2)**

Sol. Let A and B be two subsets.

For each $x \in \{1,2,3,4,5\}$, there are four possibilities :

$x \in A \cap B,\ x \in A' \cap B,\ x \in A \cap B',\ x \in A' \cap B'$

So, the number of elements in sample space $= 4^5$

Required probability

$= \frac{^5C_2 \times 3^3}{4^5}$

$= \frac{10\times 27}{2^{10}} = \frac{135}{2^9}$

5. The vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(\hat{i}-2\hat{j})=-2$, and the point (1, 0, 2) is :

(1) $\vec{r}\cdot(\hat{\imath}-7\hat{\jmath}+3\hat{k})=\frac{7}{3}$

(2) $\vec{r}\cdot(\hat{\imath}+7\hat{\jmath}+3\hat{k})=7$

(3) $\vec{r}\cdot(3\hat{\imath}+7\hat{\jmath}+3\hat{k})=7$

(4) $\vec{r}\cdot(\hat{\imath}+7\hat{\jmath}+3\hat{k})=\frac{7}{3}$

Ans. **(2)**

Sol. Family of planes passing through intersection of planes is

$\{\vec{r}\cdot(\hat{\imath}+\hat{\jmath}+\hat{k})-1\}+\lambda\{\vec{r}\cdot(\hat{\imath}-2\hat{\jmath})+2\}=0$

The above curve passes through $\hat{\imath}+2\hat{k}$,

$(3-1)+\lambda(1+2)=0 \Rightarrow \lambda=-\frac{2}{3}$

Hence, equation of plane is

$3\{\vec{r}\cdot(\hat{\imath}+\hat{\jmath}+\hat{k})-1\}-2\{\vec{r}\cdot(\hat{\imath}-2\hat{\jmath})+2\}=0$

$\Rightarrow \vec{r}\cdot(\hat{\imath}+7\hat{\jmath}+3\hat{k})=7$

TRICK: Only option (2) satisfies the point (1, 0, 2)

6. If P is a point on the parabola $y=x^2+4$ which is closest to the straight line $y=4x-1$, then the co-ordinates of P are :

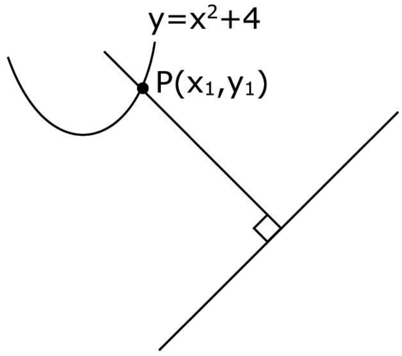
(1) (–2, 8) (2) (1, 5) (3) (3, 13) (4) (2, 8)

Ans. **(4)**

Sol. Tangent at P is parallel to the given line.

$\frac{dy}{dx}|_P = 4$

$\Rightarrow 2x_1 = 4$



$\Rightarrow x_1 = 2$

Required point is (2, 8)

7. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices $(a,c),(2,b)$ and (a,b) be $\left(\frac{10}{3},\frac{7}{3}\right)$. If α,β are the roots of the equation $ax^2+bx+1=0$, then the value of $\alpha^2+\beta^2-\alpha\beta$ is :

(1) $\frac{71}{256}$ (2) $-\frac{69}{256}$ (3) $\frac{69}{256}$ (4) $-\frac{71}{256}$

Ans. (4)

Sol. 2b = a + c

$\frac{2a+2}{3}=\frac{10}{3}$ and $\frac{2b+c}{3}=\frac{7}{3}$

$\Rightarrow a = 4$ $\left.\begin{matrix}2b+c=7\\2b-c=4\end{matrix}\right\}$, solving

$b=\frac{11}{4}$ and $c=\frac{3}{2}$

$\therefore$ Quadratic equation is $4x^2+\frac{11}{4}x+1=0$

$\therefore$ The value of $(\alpha+\beta)^2-3\alpha\beta=\frac{121}{256}-\frac{3}{4}=-\frac{71}{256}$

8. The value of the integral, $\int_1^3\left[x^2-2x-2\right]dx,$ where [x] denotes the greatest integer less than or equal to x, is :

(1) –4 (2) –5 (3) $-\sqrt{2}-\sqrt{3}-1$ (4) $-\sqrt{2}-\sqrt{3}+1$

Ans. (3)

Sol. $I=\int_1^3 -3dx+\int_1^3 [(x-1)^2]dx$

Put $x-1=t$; $dx=dt$

$I=(-6)+\int_0^2 [t^2]dt$

$I=-6+\int_0^1 0dt+\int_1^{\sqrt{2}} 1dt+\int_{\sqrt{2}}^{\sqrt{3}} 2dt+\int_{\sqrt{3}}^{2} 3dt$

$\mathrm{I} = -6 + (\sqrt{2} - 1) + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$

$\mathrm{I} = -1 - \sqrt{2} - \sqrt{3}$

9. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4 \end{cases}$$

Let $A = \{x \in \mathbf{R} : f \text{ is increasing}\}$. Then A is equal to :

(1) $(-5, -4) \cup (4, \infty)$ (2) $(-5, \infty)$

(3) $(-\infty, -5) \cup (4, \infty)$ (4) $(-\infty, -5) \cup (-4, \infty)$

Ans. **(1)**

Sol. $$f'(x) = \begin{cases} -55 & ; \quad x < -5 \\ 6(x^2 - x - 20) & ; \quad -5 < x < 4 \\ 6(x^2 - x - 6) & ; \quad x > 4 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} -55 & ; \quad x < -5 \\ 6(x - 5)(x + 4) & ; \quad -5 < x < 4 \\ 6(x - 3)(x + 2) & ; \quad x > 4 \end{cases}$$

Hence, $f(x)$ is monotonically increasing in $(-5, -4) \cup (4, \infty)$

10. If the curve $y = ax^2 + bx + c, x \in \mathbf{R}$ passes through the point $(1, 2)$ and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are :

(1) $a = 1, b = 1, c = 0$ (2) $a = -1, b = 1, c = 1$

(3) $a = 1, b = 0, c = 1$ (4) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$

Ans. **(1)**

Sol. $2 = a + b + c$

$\frac{dy}{dx} = 2ax + b, \ \left(\frac{dy}{dx}\right)_{(0,0)} = 1$

$\Rightarrow b = 1 \text{and } a + c = 1$

Since $(0, 0)$ lies on curve,

$\therefore c = 0, a = 1$

TRICK : (0,0) lies on the curve. Only option (1) has $c = 0$

11. The negation of the statement $\sim p \wedge (p \vee q)$ is :

(1) $\sim p \wedge q$ (2) $p \wedge \sim q$ (3) $\sim p \vee q$ (4) $p \vee \sim q$

Ans. **(4)**

Sol. Negation of $\sim p \wedge (p \vee q)$ is

$\sim [\sim p \wedge (p \vee q)]$

$\equiv p \vee \sim (p \vee q)$

$\equiv p \vee (\sim p \wedge \sim q)$

$\equiv (p \vee \sim p) \wedge (p \vee \sim q)$

$\equiv T \wedge (p \vee \sim q)$, where T is tautology.

$\equiv p \vee \sim q$

12. For the system of linear equations :

$x-2y=1, x-y+kz=-2, ky+4z=6, k \in \mathbf{R}$

consider the following statements :

(A) The system has unique solution if $k \neq 2, k \neq -2$.

(B) The system has unique solution if $k=-2$.

(C) The system has unique solution if $k=2$.

(D) The system has no-solution if $k=2$.

(E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are **correct** ?

(1) (B) and (E) only
(2) (C) and (D) only
(3) (A) and (D) only
(4) (A) and (E) only

Ans. **(3)**

Sol. $x-2y+0.z=1$

$x-y+kz=-2$

$0.x+ky+4z=6$

$$\Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 4-k^2$$

For unique solution, $4-k^2 \neq 0$

$$\boxed{k \neq \pm 2}$$

For $k=2$,

$x-2y+0.z=1$

$x-y+2z=-2$

$0.x + 2y + 4z = 6$

$$\Delta_x = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & 2 \\ 6 & 2 & 4 \end{vmatrix} = (-8) + 2[-20]$$

$\Rightarrow \Delta_x = -48 \neq 0$

For $k = 2, \Delta_x \neq 0$

So, for $k = 2$, the system has no solution.

13. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

(1) $x^2 + 9y^2 = 9$ (2) $2x^2 - 18y^2 = 9$

(3) $y^2 = \frac{1}{6\sqrt{3}}x$ (4) $x^2 + y^2 = 7$

Ans. **(1)**

Sol. Tangent to $x^2 + 9y^2 = 9$ at point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$ is $x\left(\frac{3\sqrt{3}}{2}\right) + 9y\left(\frac{1}{2}\right) = 9$

$\Rightarrow 3\sqrt{3}x + 9y = 18 \Rightarrow x + \sqrt{3}y = 2\sqrt{3}$

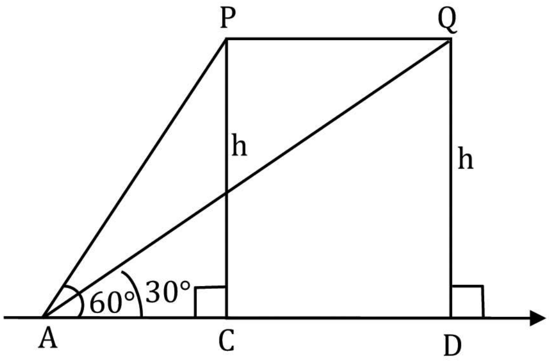
$\Rightarrow$ Option (1) is true.

14. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is :

(1) $1200\sqrt{3}$ m (2) $1800\sqrt{3}$ m (3) $3600\sqrt{3}$ m (4) $2400\sqrt{3}$ m

Ans. **(1)**

Sol.



v = 432 × $\frac{1000}{60\times60}$ m/sec = 120 m/sec

Distance PQ = v × 20 = 2400 m

In ΔPAC

$\tan 60° = \frac{h}{AC} \quad \Rightarrow AC = \frac{h}{\sqrt{3}}$

In ΔAQD

$\tan 30° = \frac{h}{AD} \quad \Rightarrow AD = \sqrt{3}h$

AD = AC + CD

$\Rightarrow \sqrt{3}h = \frac{h}{\sqrt{3}} + 2400 \Rightarrow \frac{2h}{\sqrt{3}} = 2400$

$\Rightarrow$ h = $1200\sqrt{3}$ m

15. For the statements p and q, consider the following compound statements :

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $\left((p \vee q)\right) \wedge \sim p) \rightarrow q$

Then which of the following statements is **correct** ?

(1) (a) is a tautology but not (b)

(2) (a) and (b) both are not tautologies.

(3) (a) and (b) both are tautologies.

(4) (b) is a tautology but not (a).

Ans. **(3)**

Sol. (a)

p	q	$\sim q$	$p \to q$	$\sim q \wedge (p \to q)$	$\sim p$	$(\sim q) \wedge (p \to q) \to \sim p$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

(a) is tautology.

(b)

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	$\left((p \vee q) \wedge \sim p\right) \to q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

(b) is tautology.

∴ (a) and (b) both are tautologies.

16. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $\left(A^2 B^2 - B^2 A^2\right) X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has :

(1) a unique solution
(2) exactly two solutions
(3) infinitely many solutions
(4) no solution

Ans. **(3)**

Sol. $A^T = A$, $B^T = -B$

Let $A^2B^2 - B^2A^2 = P$

$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T$

$= B^2A^2 - A^2B^2$

⇒ P is a skew-symmetric matrix.

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

∴ $ay + bz = 0$ ...(1)

$-ax + cz = 0$ ...(2)

$-bx - cy = 0$ ...(3)

From equation (1), (2), (3)

$\Delta = 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$

∴ System of equations has infinite number of solutions.

17. If $n \geq 2$ is a positive integer, then the sum of the series

${}^{n+1}C_2 + 2\left({}^{2}C_2 + {}^{3}C_2 + {}^{4}C_2 + \ldots + {}^{n}C_2\right)$ is :

(1) $\frac{n(n+1)^2(n+2)}{12}$ (2) $\frac{n(n-1)(2n+1)}{6}$

(3) $\frac{n(n+1)(2n+1)}{6}$ (4) $\frac{n(2n+1)(3n+1)}{6}$

Ans. (3)

Sol. ${}^{2}C_2 = {}^{3}C_3$

Let S = ${}^{3}C_3 + {}^{3}C_2 + \ldots\ldots + {}^{n}C_2 = {}^{n+1}C_3$ $(\because {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r)$

$\therefore {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3$

$= {}^{n+2}C_3 + {}^{n+1}C_3$

$= \frac{(n+2)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$

$= \frac{(n+2)(n+1)n}{6} + \frac{(n+1)(n)(n-1)}{6} = \frac{n(n+1)(2n+1)}{6}$

TRICK : Put $n = 2$ and verify the options.

18. If a curve $y = f(x)$ passes through the point (1, 2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)dx = \frac{62}{5}$?

(1) 5 (2) $\frac{62}{5}$ (3) $\frac{31}{5}$ (4) 10

Ans. (4)

Sol. $\frac{dy}{dx} + \frac{y}{x} = bx^3$

I.F. $= e^{\int \frac{dx}{x}} = x$

$\therefore yx = \int bx^4 dx = \frac{bx^5}{5} + c$

Above curve passes through (1,2).

$2 = \frac{b}{5} + c$

Also, $\int_1^2 \left(\frac{bx^4}{5} + \frac{c}{x}\right) dx = \frac{62}{5}$

$\Rightarrow \frac{b}{25} \times 32 + c \ln 2 - \frac{b}{25} = \frac{62}{5}$

$\Rightarrow c = 0$ and $b = 10$

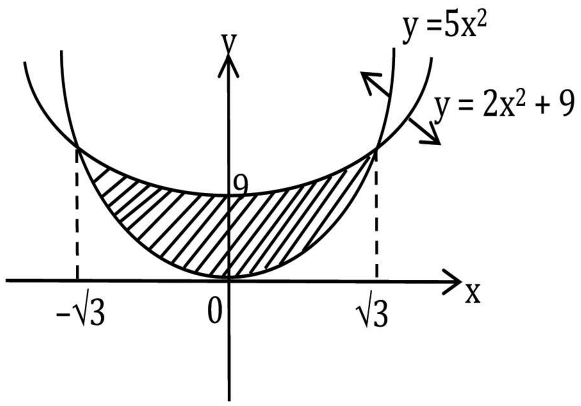
19. The area of the region : $R = \{(x, y): 5x^2 \le y \le 2x^2 + 9\}$ is :

(1) $9\sqrt{3}$ square units (2) $12\sqrt{3}$ square units

(3) $11\sqrt{3}$ square units (4) $6\sqrt{3}$ square units

Ans. (2)

Sol.



Required area

$= 2\int_0^{\sqrt{3}}(2x^2 + 9 - 5x^2)dx$

$= 2\int_0^{\sqrt{3}}(9 - 3x^2)dx$

$= 2|9x - x^3|_0^{\sqrt{3}} = 12\sqrt{3}$

20. Let $f(x)$ be a differentiable function defined on $[0, 2]$ such that $f'(x) = f'(2 - x)$ for all $x \in (0, 2)$, $f(0) = 1$ and $f(2) = e^2$. Then the value of $\int_0^2 f(x)dx$ is :

(1) $1 + e^2$ (2) $1 - e^2$ (3) $2(1 - e^2)$ (4) $2(1+e^2)$

Ans. (1)

Sol. $f'(x) = f'(2 - x)$

On integrating both sides, we get

$f(x) = -f(2 - x) + c$

Put $x = 0$

$f(0) + f(2) = c$

$\Rightarrow c = 1 + e^2$

$\Rightarrow f(x) + f(2 - x) = 1 + e^2$

$I = \int_0^2 f(x)dx = \int_0^1 \{f(x) + f(2-x)\}dx = 1 + e^2$

Section B

1. The number of the real roots of the equation $(x+1)^2 + |x-5| = \frac{27}{4}$ is ______.

Ans. **2**

Sol. For $x \geq 5$,

$(x+1)^2 + (x-5) = \frac{27}{4}$

$\Rightarrow x^2 + 3x - 4 = \frac{27}{4}$

$\Rightarrow x^2 + 3x - \frac{43}{4} = 0$

$\Rightarrow 4x^2 + 12x - 43 = 0$

$x = \frac{-12 \pm \sqrt{144+688}}{8}$

$x = \frac{-12 \pm \sqrt{832}}{8} = \frac{-12 \pm 28.8}{8}$

$= \frac{-3 \pm 7.2}{2}$

$= \frac{-3+7.2}{2}, \frac{-3-7.2}{2}$ (therefore, no solution)

For $x < 5$,

$(x+1)^2 - (x-5) = \frac{27}{4}$

$\Rightarrow x^2 + x + 6 - \frac{27}{4} = 0$

$\Rightarrow 4x^2 + 4x - 3 = 0$

$x = \frac{-4 \pm \sqrt{16+48}}{8}$

$x = \frac{-4 \pm 8}{8} \Rightarrow x = -\frac{12}{8}, \frac{4}{8}$

$\therefore$ 2 real roots.

2. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is ______.

Ans. **31650**

Sol.

C→1 $\quad 9\left[\begin{array}{l}A\\B\end{array}\right.$

C→2 $\quad 8\left[\begin{array}{l}A\\B\end{array}\right.$

C→3 $\quad 7\left[\begin{array}{l}A\\B\end{array}\right.$

Number of ways

$= {}^{10}C_1\,[2^9-2] + {}^{10}C_2\,[2^8-2] + {}^{10}C_3\,[2^7-2]$

$= 2^7[{}^{10}C_1\times4 + {}^{10}C_2\times2 + {}^{10}C_3] - 20 - 90 - 240$

$= 128\,[40 + 90 + 120] - 350$

$= (128 \times 250) - 350$

$= 10[3165] = 31650$

3. If $a+\alpha=1, b+\beta=2$ and $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}, x\neq 0$, then the value of the expression $\dfrac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is ______.

Ans. **2**

Sol. $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}$ ...(i)

Replace x by $\frac{1}{x}$

$af\left(\frac{1}{x}\right)+\alpha f(x)=\frac{b}{x}+\beta x$ ...(ii)

(i) + (ii)

$(a+\alpha)\left[f(x)+f\left(\frac{1}{x}\right)\right]=\left(x+\frac{1}{x}\right)(b+\beta)$

$\Rightarrow \dfrac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}=\frac{2}{1}=2$

4. If the variance of 10 natural numbers $1, 1, 1, \ldots, 1, k$ is less than 10, then the maximum possible value of k is ______.

Ans. 11

Sol. $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$\Rightarrow \sigma^2 = \frac{(9+k^2)}{10} - \left(\frac{9+k}{10}\right)^2 < 10$

$\Rightarrow 10(9+k^2) - (81+k^2+18k) < 1000$

$\Rightarrow 90 + 10k^2 - k^2 - 18k - 81 < 1000$

$\Rightarrow 9k^2 - 18k + 9 < 1000$

$\Rightarrow (k-1)^2 < \frac{1000}{9} \Rightarrow k - 1 < \frac{10\sqrt{10}}{3}$

$\Rightarrow k < \frac{10\sqrt{10}}{3} + 1$

Maximum possible integral value of k is 11.

5. Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is ______.

Ans. 1

Sol. $\frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$

$\frac{x-\lambda}{2} = \frac{y-\frac{1}{2}}{1} = \frac{z}{-1}$ ...(1) Point on line $= \left(\lambda, \frac{1}{2}, 0\right)$

$\frac{x}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1}$ ...(2) Point on line $= (0, -2\lambda, \lambda)$

Distance between skew lines $= \frac{[\vec{a}_2 - \vec{a}_1 \ \ \vec{b}_1 \ \ \vec{b}_2]}{|\vec{b}_1 \times \vec{b}_2|}$

$$\frac{\begin{vmatrix} \lambda & \frac{1}{2}+2\lambda & -\lambda \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}{\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}\right|}$$

$= \frac{\left|-5\lambda - \frac{3}{2}\right|}{\sqrt{14}} = \frac{\sqrt{7}}{2\sqrt{2}}$ (Given)

$\Rightarrow |10\lambda + 3| = 7 \Rightarrow \lambda = -1$ as λ is an integer.

$\Rightarrow |\lambda| = 1$

6. Let $i=\sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}}+\frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}}=k$, and $n=[|k|]$ be the greatest integral part of |k|. Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to _____.

Ans. 310

Sol. $\frac{\left(2e^{i\frac{2\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^{24}}+\frac{\left(2e^{i\frac{\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^{24}}$

$=\frac{2^{21}.e^{i14\pi}}{2^{12}.e^{-i6\pi}}+\frac{2^{21}(e^{i7\pi})}{2^{12}(e^{i6\pi})}$

$=2^9e^{i(20\pi)}+2^9e^{i\pi}$

$=2^9+2^9(-1)=0=k$

$\therefore n=0$

$\sum_{j=0}^{5}(j+5)^2-\sum_{j=0}^{5}(j+5)$

$=[5^2+6^2+7^2+8^2+9^2+10^2]-[5+6+7+8+9+10]$

$=[(1^2+2^2+\ldots+10^2)-(1^2+2^2+3^2+4^2)]-[(1+2+3+\ldots+10)-(1+2+3+4)]$

$=(385-30)-[55-10]$

$=355-45=310$

7. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point $(-5,0)$. If the locus of the point P is a circle of radius r, then $4r^2$ is equal to _______.

Ans. 56.25

Sol. Let P be (h,k), A(5, 0) and B(-5, 0)

Given PA = 3PB

$\Rightarrow PA^2=9PB^2$

$\Rightarrow (h-5)^2+k^2=9[(h+5)^2+k^2]$

$\Rightarrow 8h^2+8k^2+100h+200=0$

$\therefore$ Locus of P is $x^2+y^2+\left(\frac{25}{2}\right)x+25=0$

Centre $\equiv\left(\frac{-25}{4},0\right)$

$\therefore r^2=\left(\frac{-25}{4}\right)^2-25$

$= \frac{625}{16} - 25$

$= \frac{225}{16}$

$\therefore 4r^2 = 4 \times \frac{225}{16} = \frac{225}{4} = 56.25$

8. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \ge r \ge 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum

$\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to_______.

Ans. *

Sol. BONUS

$(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1x + {}^{10}C_2x^2 + \ldots\ldots + {}^{10}C_{10}x^{10}$

$(1+x)^{15} = {}^{15}C_0 + {}^{15}C_1x + \ldots.{}^{15}C_{k-1}x^{k-1} + {}^{15}C_kx^k + {}^{15}C_{k+1}x^{k+1} + \ldots\ldots{}^{15}C_{15}x^{15}$

$\sum_{i=0}^{k}(10C_i)(15C_{k-i}) = {}^{10}C_0.\,{}^{15}C_k + {}^{10}C_1.{}^{15}C_{k-1} + \ldots..+{}^{10}C_k.{}^{15}C_0$

Coefficient of x_k in $(1+x)^{25}$

$= {}^{25}C_k$

$\sum_{i=0}^{k+1}(12C_i)(13C_{k+1-i}) = {}^{12}C_0.{}^{13}C_{k+1} + {}^{12}C_1.\,{}^{13}C_k + \ldots\ldots + {}^{12}C_{k+1}.{}^{13}C_0$

Coefficient of x^{k+1} in $(1+x)^{25}$

$= {}^{25}C_{k+1}$

${}^{25}C_k + {}^{25}C_{k+1} = {}^{26}C_{k+1}$

By the given definition of $\binom{n}{r}$, k can be as large as possible.

9. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is________.

Ans. **3**

Sol. a, ar, ar^2, ar^3

$a + ar + ar^2 + ar^3 = \frac{65}{12}$ ....(1)

$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$

$\Rightarrow \frac{1}{a}\left(\frac{r^3+r^2+r+1}{r^3}\right) = \frac{65}{18}$ ....(2)

$\frac{(1)}{(2)}$, we get

$a^2r^3 = \frac{18}{12} = \frac{3}{2}$

Also, $a^3r^3 = 1 \Rightarrow a\left(\frac{3}{2}\right) = 1 \Rightarrow a = \frac{2}{3}$

$\frac{4}{9}r^3 = \frac{3}{2} \Rightarrow r^3 = \frac{3^3}{2^3} \Rightarrow r = \frac{3}{2}$

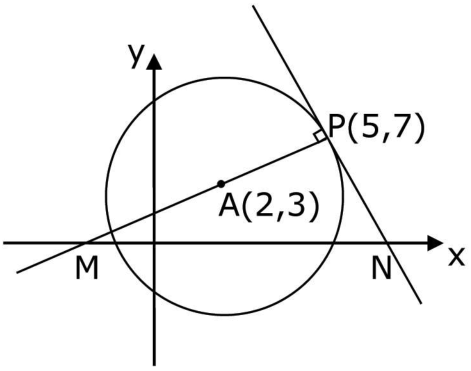
$\alpha = ar^2 = \frac{2}{3}.\left(\frac{3}{2}\right)^2 = \frac{3}{2}$

$\therefore 2\alpha = 3$

10. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2+(y-3)^2=25$ at the point (5, 7) is A, then 24A is equal to ______.

Ans.

Sol.



Equation of normal at P is

$(y-7) = \left(\frac{7-3}{5-2}\right)(x-5)$

$\Rightarrow$ 3y – 21 = 4x – 20

$\Rightarrow$ 4x – 3y + 1 = 0

$\Rightarrow M$ is $\left(-\frac{1}{4}, 0\right)$

Equation of tangent at P is

$(y-7) = -\frac{3}{4}(x-5)$

$\Rightarrow$ 4y – 28 = –3x + 15

$\Rightarrow$ 3x + 4y = 43

$\Rightarrow N$ is $\left(\frac{43}{3}, 0\right)$

The question is **wrong**. The normal cuts at a point on the negative axis.

Section - A

1. Zener breakdown occurs in a p-n junction having p and n both:

(1) lightly doped and have wide depletion layer.

(2) heavily doped and have narrow depletion layer.

(3) heavily doped and have wide depletion layer.

(4) lightly doped and have narrow depletion layer.

Ans. (2)

Sol. The Zener breakdown occurs in the heavily doped p-n junction diode. Heavily doped p-n junction diodes have narrow depletion region. The narrow depletion layer width leads to a high electric field which causes the p-n junction breakdown.

2. According to Bohr atom model, in which of the following transitions will the frequency be maximum?

(1) n=2 to n=1 (2) n=4 to n =3

(3) n=5 to n=4 (4) n=3 to n=2

Ans. (1)

Sol.

ΔE=hf

$\Delta E = 13.4Z^2[\frac{1}{n^2} - \frac{1}{n_1^2}]$

Since, ΔE is maximum for the transition from n = 2 to n = 1

f is more for transition from n = 2 to n = 1.

3. An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be:

(1) 10^{-2} nm (2) 10^{-3} nm

(3) 10^{-4} nm (4) 10^{-1} nm

Ans. **(2)**

Sol. The minimum wavelength of photon will correspond to the maximum energy due to accelerating by V volts in the tube.

$\lambda_{min} = \frac{hc}{eV}$

$\lambda_{min} = \frac{1240 nm-eV}{1.24\times10^6}$

$\lambda_{min} = 10^{-3}\ nm$

4. On the basis of kinetic theory of gases, the gas exerts pressure because its molecules:

(1) suffer change in momentum when impinge on the walls of container.

(2) continuously stick to the walls of container.

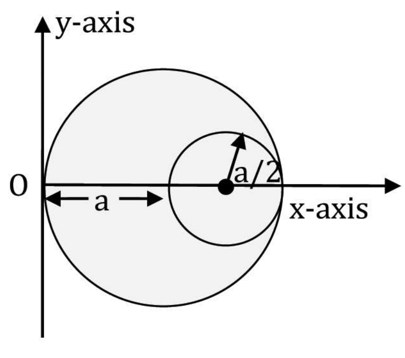
(3) continuously lose their energy till it reaches wall.

(4) are attracted by the walls of container.

Ans. **(1)**

Sol. Based on kinetic theory of gases, molecules suffer change in momentum when impinge on the walls of container. Due to this they exert a force resulting in exerting pressure on the walls of the container.

5. A circular hole of radius $(\frac{a}{2})$ is cut out of a circular disc of radius 'a' shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be:

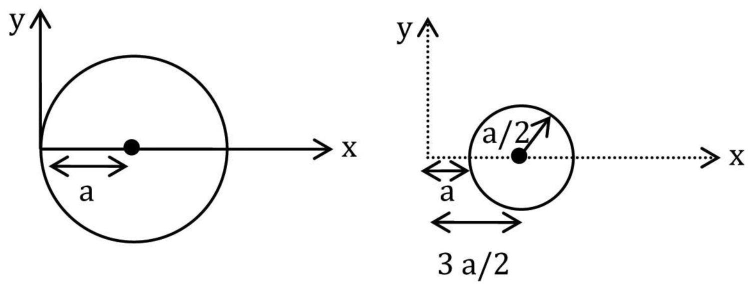


(1) $\frac{10}{11}a$ (2) $\frac{2}{3}a$

(3) $\frac{1}{6}a$ (4) $\frac{5}{6}a$

Ans. **(4)**

Sol. Let σ be the surface mass density of disc.



$X_{com} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$

Where m = σ πr²

$X_{com} = \frac{(\sigma\times\pi a^2\times a)-(\sigma\frac{\pi a^2}{4}\times\frac{3a}{2})}{\sigma\pi a^2-\frac{\sigma\pi a^2}{4}}$

$X_{com} = \frac{a-3\frac{a}{8}}{1-\frac{1}{4}}$

$X_{com} = \frac{\frac{5a}{8}}{\frac{3}{4}}$

$X_{com} = \frac{5a}{6}$

6. Given below are two statements:

Statement I: PN junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.

Statement II: In the study of transistor, the amplification factor β indicates ratio of the collector current to the base current.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement I is false but Statement II is true.

(2) Both Statement I and Statement II are true

(3) Statement I is true but Statement II is false.

(4) Both Statement I and Statement II are false

Ans. **(1)**

Sol. **S-1**

Statement 1 is false because in case of two discrete back to back connected diodes, there are four doped regions instead of three and there is nothing that resembles a thin base region between an emitter and a collector.

S-2

Statement-2 is true, as

$\beta = \frac{I_C}{I_B}$

7. When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is:

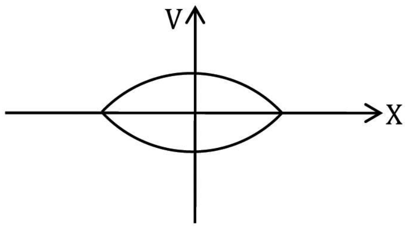
(1) elliptical (2) parabolic

(3) straight line (4) circular

Ans. **(1)**

Sol. We know that in SHM;

$V = \omega\sqrt{A^2 - x^2}$



Elliptical

Alternate:

x = A sinωt ⇒ sinωt = $\frac{x}{A}$

v = Aωcosωt ⇒ cosωt = $\frac{v}{A\omega}$

Hence $(\frac{x}{A})^2 + (\frac{v}{A\omega})^2 = 1$

which is the equation of a ellipse.

8. Match List – I with List – II.

List – I	List - II
(a) Source of microwave frequency	(i) Radioactive decay of nucleus
(b) Source of infrared frequency	(ii) Magnetron
(c) Source of Gamma Rays	(iii) Inner shell electrons
(d) Source of X-rays	(iv) Vibration of atoms and molecules
	(v) LASER
	(vi) RC circuit

Choose the correct answer from the options given below:

(1) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(2) (a)-(vi), (b)-(iv), (c)-(i), (d)-(v)

(3) (a)-(ii), (b)-(iv), (c)-(vi), (d)-(iii)

(4) (a)-(vi), (b)-(v), (c)-(i), (d)-(iv)

Ans. (1)

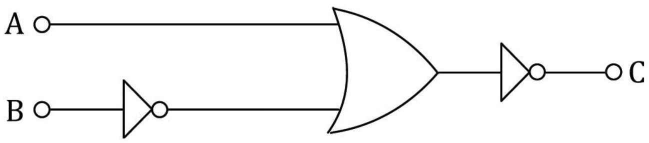
Sol. (a) Source of microwave frequency – (ii) Magnetron

(b) Source of infra-red frequency – (iv) Vibration of atom and molecules

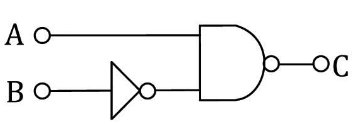
(c) Source of gamma ray – (i) Radioactive decay of nucleus

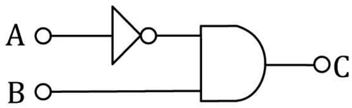
(d) Source of X-ray – (iii) inner shell electron

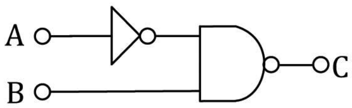
9.

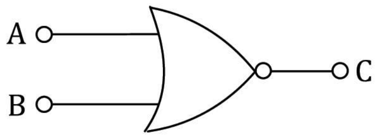


The logic circuit shown above is equivalent to :

(1)


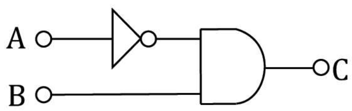
(2)


(3)


(4)


Ans. (2)

Sol.



$C = \overline{A + \overline{B}} = \overline{A}.\overline{\overline{B}} = \overline{A}.B$

$C = \overline{A}.B$

10. If the source of light used in a Young's double slit experiment is changed from red to violet:

(1) the fringes will become brighter.

(2) consecutive fringe lines will come closer.

(3) the central bright fringe will become a dark fringe.

(4) the intensity of minima will increase.

Ans. **(2)**

Sol. $\beta = \frac{\lambda D}{d}$

As $\lambda_v < \lambda_R$

$\Rightarrow \beta_v < \beta_R$

⇒ Consecutive fringe line will come closer.

11. A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator?
[Use $g=\frac{GM}{R^2}$ = 9.8 ms^{-2} and radius of earth, R = 6400 km.]

(1)49 N

(2) 49.83 N

(3) 49.17 N

(4) 48.83 N

Ans. **(4)**

Sol. At North Pole, weight

Mg = 49

Now, at equator

$g' = g - \omega^2 R$

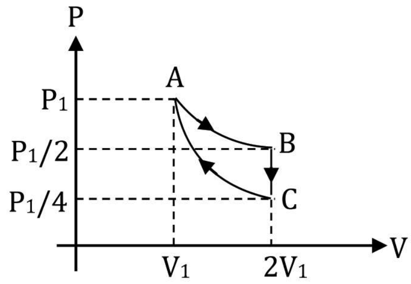
$\Rightarrow Mg' = M(g - \omega^2 R)$

⇒ weight will be less than Mg at equator.

Alter:

g is maximum at the poles.
Hence from options only (2) has lesser value than 49N.

12. If one mole of an ideal gas at (P_1, V_1) is allowed to expand reversibly and isothermally (A to B) its pressure is reduced to one-half of the original pressure (see figure). This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value (B→C). Then it is restored to its initial state by a reversible adiabatic compression (C to A). The net work done by the gas is equal to :



(1)0 (2)-$\frac{RT}{2(\gamma-1)}$

(3) $RT[ln2 - \frac{1}{2(\gamma-1)}]$ (4) RT ln2

Ans. **(3)**

Sol. AB → Isothermal process

W_{AB} = nRT ln2 = RT ln2

BC → Isochoric process

W_{BC} = 0

CA → Adiabatic process

$$W_{CA} = \frac{P_1V_1 - \frac{P_1}{4}X2V_1}{1-\gamma} = \frac{P_1V_1}{2(1-\gamma)} = \frac{RT}{2(1-\gamma)}$$

$$W_{ABCA} = RT\ ln2 + \frac{RT}{2(1-\gamma)}$$

$$= RT[ln2 - \frac{1}{2(\gamma-1)}]$$

13. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of 'L' is 1.0 m from meter scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of 'g' will be:

(1)1.33 % (2)1.30 %

(3)1.13 % (4)1.03 %

Ans. **(3)**

Sol. $T = 2\pi\sqrt{\frac{L}{g}}$

$$T^2 = 4\pi^2[\frac{L}{g}]$$

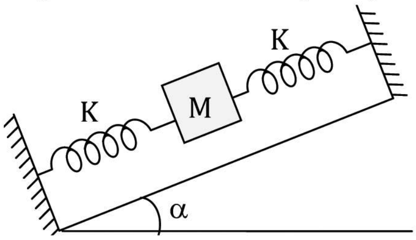
$$g = 4\pi^2[\frac{L}{T^2}]$$

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

$= [\frac{1mm}{1m} + \frac{2(10\times10^{-3})}{1.95}] \times 100$

= 1.13 %

14. In the given figure, a body of mass M is held between two massless springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant k, the frequency of oscillation of given body is:



(1) $\frac{1}{2\pi}\sqrt{\frac{2K}{Mgsin\alpha}}$

(2) $\frac{1}{2\pi}\sqrt{\frac{K}{Mgsin\alpha}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{2K}{M}}$

(4) $\frac{1}{2\pi}\sqrt{\frac{K}{2M}}$

Ans. **(1)**

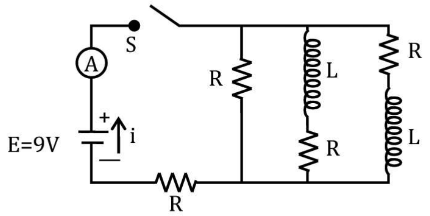
Sol. Equivalent K = K + K = 2K

Now, $T = 2\pi\sqrt{\frac{M}{K_{eq}}}$

$\Rightarrow T = 2\pi\sqrt{\frac{M}{2K}}$

$\therefore f = \frac{1}{2\pi}\sqrt{\frac{2K}{M}}$

15. Figure shows a circuit that contains four identical resistors with resistance R = 2.0 Ω. Two identical inductors with inductance L = 2.0 mH and an ideal battery with emf E = 9.V. The current '*i*' just after the switch 's' is closed will be:



(1) 9A

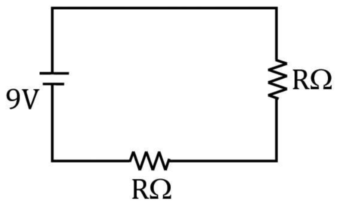
(2) 3.0 A

(3) 2.25 A

(4) 3.37 A

Ans. **(3)**

Sol. Just when switch S is closed, inductor will behave like an infinite resistance. Hence, the circuit will be like



Given: V = 9 V

From V = IR

$I = \frac{V}{R}$

$R_{eq.}$ = 2+2 = 4 Ω

$i = \frac{9}{4}$ = 2.25 A

16. The de Broglie wavelength of a proton and α-particle are equal. The ratio of their velocities is:

(1) 4:2
(2) 4:1
(3) 1:4
(4) 4:3

Ans. **(2)**

Sol. From De-Broglie's wavelength: -

$\lambda = \frac{h}{mv}$

Given $\lambda_P = \lambda_\alpha$

$v \propto \frac{1}{m}$

$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4m_p}{m_p} = \frac{4}{1}$

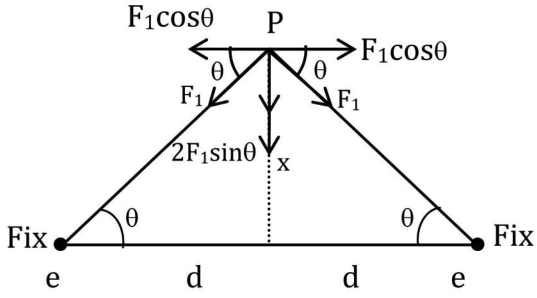
17. Two electrons each are fixed at a distance '2d'. A third charge proton placed at the midpoint is displaced slightly by a distance x (x<<d) perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency:

(m = mass of charged particle)

(1) $(\frac{q^2}{2\pi\varepsilon_0 md^3})^{\frac{1}{2}}$
(2) $(\frac{\pi\varepsilon_0 md^3}{2q^2})^{\frac{1}{2}}$
(3) $(\frac{2\pi\varepsilon_0 md^3}{q^2})^{\frac{1}{2}}$
(4) $(\frac{2q^2}{\pi\varepsilon_0 md^3})^{\frac{1}{2}}$

Ans. **(1)**

Sol.



Restoring force on proton: -

$F_r = 2F_1 \sin\theta$ where $F_1 = \frac{kq^2}{(d^2+x^2)}$

$F_r = \frac{2Kq^2x}{[d^2+x^2]^{\frac{3}{2}}}$

x <<< d

$F_r = \frac{2kq^2x}{d^3} = \frac{q^2x}{2\pi\varepsilon_0 d^3} = kx$

$K = \frac{q^2}{2\pi\varepsilon_0 d^3}$

Angular Frequency: -

$\omega = \sqrt{\frac{k}{m}}$

$\omega = \sqrt{\frac{q^2}{2\pi\varepsilon_0 m d^3}}$

18. A soft ferromagnetic material is placed in an external magnetic field. The magnetic domains:

(1) decrease in size and changes orientation.

(2) may increase or decrease in size and change its orientation.

(3) increase in size but no change in orientation.

(4) have no relation with external magnetic field.

Ans. (2)

Sol. Atoms of ferromagnetic material in unmagnetized state form domains inside the ferromagnetic material. These domains have large magnetic moment of atoms. In the absence of magnetic field, these domains have magnetic moment in different directions. But when the magnetic field is applied, domains aligned in the direction of the field grow in size and those aligned in the direction opposite to the field reduce in size and also its orientation changes.

19. Which of the following equations represents a travelling wave?

(1) $y = Ae^{-x^2}(vt + \theta)$

(2) $y = Asin(15x - 2t)$

(3) $y = Ae^{x}cos(\omega t - \theta)$

(4) $y = Asinxcos\omega t$

Ans. **(2)**

Sol. Y = F(x,t)

For travelling wave y should be linear function of x and t and they must exist as (x±vt)

Y = A sin (15x-2t) which is a linear function in x and t.

20. A particle is projected with velocity v_0 along x-axis. A damping force is acting on the particle which is proportional to the square of the distance from the origin i.e. ma = $-\alpha x^2$. The distance at which the particle stops:

(1) $(\frac{2v_0}{3\alpha})^{\frac{1}{3}}$

(2) $(\frac{3v_0^2}{2\alpha})^{\frac{1}{2}}$

(3) $(\frac{3v_0^2}{2\alpha})^{\frac{1}{3}}$

(4) $(\frac{2v_0^2}{3\alpha})^{\frac{1}{2}}$

Ans. **Bonus**

Sol. $a = \frac{vdv}{dx}$

$\int_{v_i}^{v_f} Vdv = \int_{x_i}^{x_f} adx$

Given: - $v_i = v_0$

$V_f = 0$

$X_i = 0$

$X_f = x$

From Damping Force: a = $-\frac{\alpha x^2}{m}$

$\int_{V_0}^{O} VdV = -\int_{o}^{x} \frac{\alpha x^2}{m} dx$

$-\frac{v_0^2}{2} = \frac{-\alpha}{m}[\frac{x^3}{3}]$

$x = [\frac{3mv_0^2}{2\alpha}]^{\frac{1}{3}}$

Most suitable answer could be (3) as mass 'm' is not given in any options.

Section - B

1. A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled and subjected to the same force will be _______ cm.

Ans. **2**

Sol.

$y = \frac{Fl}{A\,\Delta l}$

$\Rightarrow \frac{F}{A} = y\frac{\Delta l}{l}$

$\Rightarrow \frac{F}{A} = y \times \frac{0.04}{l}$ ...(1)

When length & diameter is doubled.

$\Rightarrow \frac{F}{4A} = y \times \frac{\Delta l}{2l}$ ...(2)

(1) ÷ (2)

$\frac{\frac{F}{A}}{F/4A} = \frac{y\times\frac{0.04}{l}}{y\times\frac{\Delta l}{2l}}$

$4 = \frac{0.04\times 2}{\Delta l}$

$\Delta l = 0.02$

$\Delta l = 2 \times 10^{-2}$

$\therefore x = 2$

2. A cylindrical wire of radius 0.5 mm and conductivity 5×10^7 S/m is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi$ mA. The value of x is ____.

Ans. **5**

Sol. We know that

$J = \sigma E$

$\Rightarrow J = 5 \times 10^7 \times 10 \times 10^{-3}$

$\Rightarrow J = 50 \times 10^4$ A/m^2

Current flowing;

$I = J \times \pi R^2$

$I = 50\times10^4 \times\pi(0.5\times10^{-3})^2$

$I = 50\times10^4\times\pi\times0.25\times10^{-6}$

$I = 125\times10^{-3}\pi$

$X = 5$

3. Two cars are approaching each other at an equal speed of 7.2 km/hr. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be _______ Hz. [Velocity of sound in air is 340 m/s.]

Ans. **8**

Sol.

Speed = 7.2 km/h = 2 m/s

Frequency as heard by A

$f_A' = f_B(\frac{v+v_0}{v-v_s})$

$f_A' = 676(\frac{340+2}{340-2})$

$f_A' = 684Hz$

$\therefore f_{Beat} = f_A' - f_B$

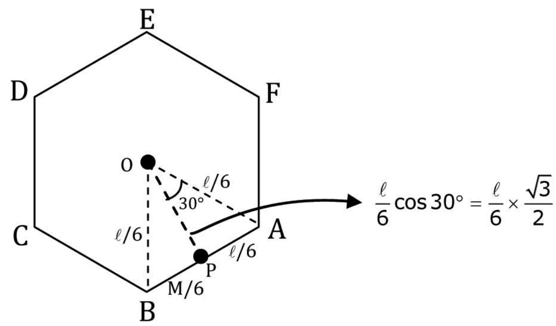
=684-676

= 8 Hz

4. A uniform thin bar of mass 6 kg and length 2.4 meter is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is _____$\times10^{-1}$ kg m^2.

Ans. **8**

Sol.



MOI of AB about P : $I_{AB_P} = \frac{\frac{M}{6}(\frac{l}{6})^2}{12}$

MOI of AB about O,

$I_{AB_O} = [\frac{\frac{M}{6}(\frac{l}{6})^2}{12} + \frac{M}{6}(\frac{l}{6}\frac{\sqrt{3}}{2})^2]$

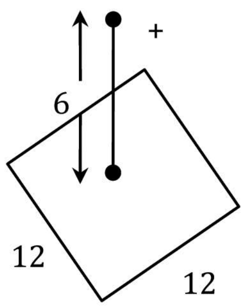
$I_{Hexagon_0} = 6I_{AB_0} = M[\frac{l^2}{12\times36} + \frac{l^2}{36} \times \frac{3}{4}]$

$= \frac{6}{100}[\frac{24\times24}{12\times36} + \frac{24\times24}{36} \times \frac{3}{4}]$

= 0.8 kg m^2

= 8×10^{-1} kg m^2

5. A point charge of +12 μC is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be _____ $\times10^3$ Nm^2/C.



Ans. **226**

Sol. Using Gauss law, it is a part of cube of side 12 cm and charge at centre,

$\phi = \frac{Q}{6\varepsilon_0} = \frac{12\mu c}{6\varepsilon_0} = 2 \times 4\pi \times 9 \times 10^9 \times 10^{-6}$

$= 226 \times 10^3$ Nm^2/C

6. Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energies $(K.E.)_A : (K.E.)_B$ will be $\frac{A}{1}$. So the value of A will be ________.

Ans. **2**

Sol. Given that, $\frac{M_1}{M_2} = \frac{1}{2}$

we know that

$K = \frac{p^2}{2M}$

$\Rightarrow \frac{K_1}{K_2} = \frac{p^2}{2M_1} \times \frac{2M_2}{p^2} \Rightarrow \frac{K_1}{K_2} = \frac{M_2}{M_1} = \frac{2}{1}$

$\Rightarrow \frac{A}{1} = \frac{2}{1} \Rightarrow \therefore A = 2$

7. The root mean square speed of molecules of a given mass of a gas at 27°C and 1 atmosphere pressure is 200 ms^{-1}. The root mean square speed of molecules of the gas at 127°C and 2 atmosphere pressure is $\frac{x}{\sqrt{3}}$ ms^{-1}. The value of x will be ________.

Ans. **400 m/s**

Sol. $V_{rms} = \sqrt{\frac{3RT_1}{M_0}}$

$200 = \sqrt{\frac{3R\times 300}{M_0}}$ ....(1)

Also, $\frac{x}{\sqrt{3}} = \sqrt{\frac{3R\times 400}{M_0}}$ ...(2)

(1) ÷ (2)

$\frac{200}{\frac{x}{\sqrt{3}}} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}}$

$\Rightarrow x = 400$ m/s

8. A series LCR circuit is designed to resonate at an angular frequency $\omega_0 = 10^5 rad/s$. The circuit draws 16W power from 120 V source at resonance. The value of resistance 'R' in the circuit is _______ Ω.

Ans. **900**

Sol. $P = \frac{V^2}{R}$

$16 = \frac{120^2}{R} \Rightarrow R = \frac{14400}{16}$

$\Rightarrow R = 900\,\Omega$

9. An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium will be ______ $\times 10^{-2}$ cm.

Ans. 667

Sol. $\epsilon_r = 2.25$

Assuming non-magnetic material ⇒ $\mu_r = 1$

Hence refractive index of the medium

$n = \sqrt{\mu_r \in_r} = \sqrt{2.25} = 1.5$

$\therefore \frac{\lambda_v}{\lambda_m} = n$

$\lambda_m = \frac{C}{f.n} = \frac{3\times 10^8}{3\times 10^9 \times 1.5} = \frac{2}{3}\times 10^{-1}\, m$

$\lambda_m = \frac{20}{3}\, cm = 667 \times 10^{-2}\, cm$

10. A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is -5 dB per km and cable length is 20 km. the power received at receiver is 10^{-x}W. The value of x is ______.

[Gain in dB = 10 $\log_{10}(\frac{P_0}{P_i})$]

Ans. 8

Sol. Power of signal transmitted: P_i = 0.1 Kw = 100w

Rate of attenuation = -5 dB/Km

Total length of path = 20 km

Total loss suffered = -5×20=-100dB

Gain in dB = 10 $\log_{10}\frac{P_0}{P_i}$

$-100=10\log_{10}\frac{P_0}{P_i}$

$\Rightarrow log_{10}\frac{P_i}{P_0} = 10$

$\Rightarrow log_{10}\frac{P_i}{P_0} = log_{10}10^{10}$

$\Rightarrow \frac{100}{P_0} = 10^{10}$

$\Rightarrow P_0 = \frac{1}{10^8} = 10^{-8}$

$\Rightarrow$ x = 8