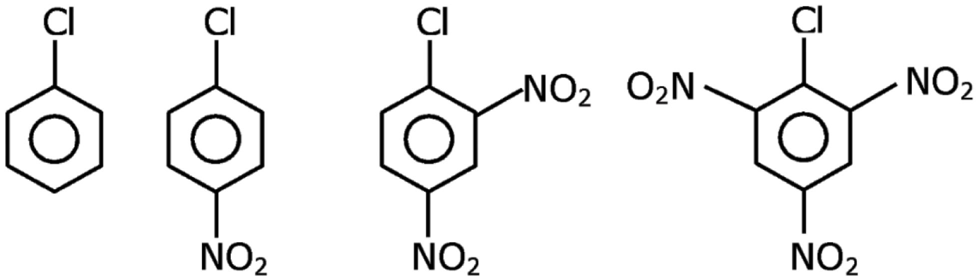
1. The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is:

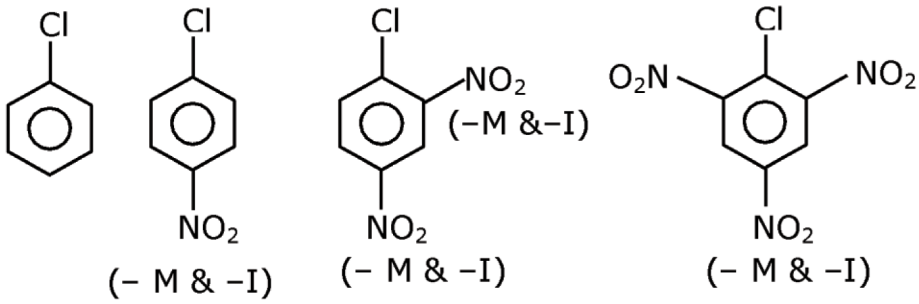


(i) (ii) (iii) (iv)

a. (iv) < (i) < (iii) < (ii)
b. (iv) < (i) < (ii) < (iii)
c. (i) < (ii) < (iii) < (iv)
d. (iv) < (iii) < (ii) < (i)

Ans (c)

Solution:



(i) (ii) (iii) (iv)

Reactivity ∝ – M group present at o/p position.

2. Match List-I with List-II

List- I (Metal)	List-II (Ores)
(a) Aluminum	(i) Siderite
(b) Iron	(ii) Calamine
(c) Copper	(iii) Kaolinite
(d) Zinc	(iv) Malachite

a. (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
b. (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
c. (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
d. (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Ans (c)

Solution:

Siderite	$FeCO_3$
Calamine	$ZnCO_3$
Kaolinite	$Si_2Al_2O_5(OH)_4$ or $Al_2O_3.2SiO_2.2H_2O$
Malachite	$CuCO_3.Cu(OH)_2$

3. Match List-I with List-II

List- I (Salt)	List-II (Flame colour wavelength)
(a) LiCl	(i) 455.5 nm
(b) NaCl	(ii) 970.8 nm
(c) RbCl	(iii) 780.0 nm
(d) CsCl	(iv) 589.2 nm

Choose the correct answer from the options given below:

a. (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
b. (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
c. (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)
d. (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

Ans (b)

Solution:

Range of visible region: -
390 nm – 760 nm

VIBGYOR

Violet - Red

LiCl	Crimson Red
NaCl	Golden yellow
RbCl	Violet
CsCl	Blue

So, LiCl which is crimson have wave length closed to red in the spectrum of visible region which is as per given data.

4. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Hydrogen is the most abundant element in the Universe, but it is not the most abundant gas in the troposphere.

Reason R: Hydrogen is the lightest element.

In the light of the above statements, choose the correct answer from the given below

(1) A is false but R is true
(2) Both A and R are true and R is the correct explanation of A
(3) A is true but R is false
(4) Both A and R are true but R is NOT the correct explanation of A

a. A is false but R is true
b. Both A and R are true and R is the correct explanation of A
c. A is true but R is false
d. Both A and R are true but R is NOT the correct explanation of A

Ans (b)

Solution:

Hydrogen is most abundant element in universe because all luminous body of universe i.e. stars & nebulae are made up of hydrogen which acts as nuclear fuel & fusion reaction is responsible for their light.

5. Given below are two statements:

Statement I: The value of the parameter ''Biochemical Oxygen Demand (BOD)'' is important for survival of aquatic life.

Statement II: The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the most appropriate answer from the options given below.

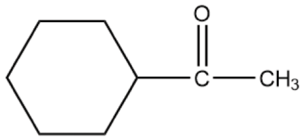
a. Both Statement I and Statement II are false
b. Statement I is false but Statement II is true
c. Statement I is true but Statement II is false
d. Both Statement I and Statement II are true

Ans (c)

For survival of aquatic life dissolved oxygen is responsible its optimum limit 6.5 ppm and optimum limit of BOD ranges from 10-20 ppm & BOD stands for biochemical oxygen demand.

6. Which one of the following carbonyl compounds cannot be prepared by addition of water on an alkyne in the presence of $HgSO_4$ and H_2SO_4?

a. $CH_3-CH_2-\overset{O}{\overset{||}{C}}-H$

b. 

c. $CH_3-\overset{O}{\overset{||}{C}}-H$

d. $CH_3-\overset{O}{\overset{||}{C}}-CH_2CH_3$

Ans (a)

Solution:

Reaction of Alkyne with $HgSO_4$ & H_2SO_4 follow as

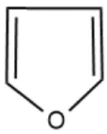
$$CH \equiv CH \xrightarrow[H_2O]{HgSO_4, H_2SO_4} CH_3CHO$$

$$CH_3-C \equiv CH \xrightarrow[H_2O]{HgSO_4, H_2SO_4} CH_3-\overset{O}{\overset{||}{C}}-CH_3$$

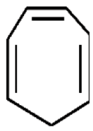
Hence, by this process preparation of CH_3CH_2CHO can't be possible.

7. Which one of the following compounds is non-aromatic?

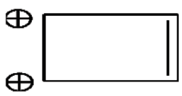
a.



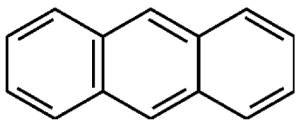
b.



c.

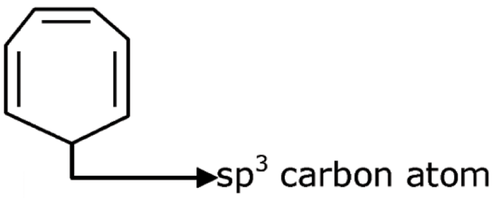


d.



Ans (b)

Solution:



(Not planar)

Hence, it is non-aromatic.

8. The incorrect statement among the following is:
   a. $VOSO_4$ is a reducing agent
   b. Red color of ruby is due to the presence of CO^{3+}
   c. Cr_2O_3 is an amphoteric oxide
   d. RuO_4 is an oxidizing agent

Ans (b)
Solution:

Red color of ruby is due to presence of CrO_3 or Cr^{+6} not CO^{3+}

9. According to Bohr's atomic theory:
   (a) Kinetic energy of electron is $\propto \frac{Z^2}{n^2}$
   (b) The product of velocity (v) of electron and principal quantum number (n). 'v_n' $\propto Z^2$
   (c) Frequency of revolution of electron in an orbit is $\propto \frac{Z^3}{n^3}$
   (d) Coulombic force of attraction on the electron is $\propto \frac{Z^3}{n^4}$

   Choose the most appropriate answer from the options given below:
   a. (c) only
   b. (a) and (d) only
   c. (a) only
   d. (a), (c) and (d) only

Ans (b)

Solution:

(a) $KE = -TE = 13.6 \times \frac{Z^2}{n^2}\ eV$

$$KE \propto \frac{Z^2}{n^2}$$

(b) $V = 2.188 \times 10^6 \times \frac{Z}{n}\ m/s$

So, $V_n \propto Z$

Frequency $= \frac{V}{2\pi r}$

$$F \propto \frac{Z^2}{n^3} \left[\therefore r \propto \frac{n^2}{z} \text{ and } v \propto \frac{Z}{n}\right]$$

(d) Force $\propto \frac{Z^2}{r^2}$

So, F $\propto \frac{Z^3}{n^4}$

So, only statement (A) is correct.

10. Match List-I with List-II

List- I	List-II
(a) Valium	(i) Antifertility drug
(b) Morphine	(ii) Pernicious anaemia
(c) Norethindrone	(iii) Analgesic
(d) Vitamin B12	(iv) Tranquilizer

a. (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
b. (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)
c. (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
d. (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

Ans (d)
Solution:

(a) Valium	(iv) Tranquilizer
(b) Morphine	(iii) Analgesic
(c) Norethindrone	(i) Antifertility drug
(d) Vitamin B12	(ii) Pernicious anemia

11. The Correct set from the following in which both pairs are in correct order of melting point is
a. LiF > LiCl ; NaCl > MgO
b. LiF > LiCl ; MgO > NaCl
c. LiCl > LiF ; NaCl > MgO
d. LiCl > LiF ; MgO > NaCl

Ans (b)

Solution:
Generally

M.P. $\propto$ Lattice energy = $\frac{KQ_1Q_2}{r^+ + r^-}$

$\propto$ (packing efficiency)

12. The calculated magnetic moments (spin only value) for species $[FeCl_4]^{2-}$, $[Co(C_2O_4)_3]^{3-}$ and MnO_4^{2-} respectively are:

a. 5.92, 4.90 and 0 BM
b. 5.82, 0 and 0 BM
c. 4.90, 0 and 1.73 BM
d. 4.90, 0 and 2.83 BM

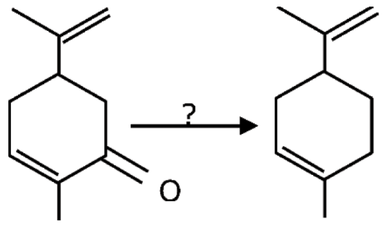
Ans (c)
Solution:
$[FeCl_4]^{2-}$

Fe^{2+} $3d^6$ → 4 unpaired electrons. as Cl^- in a weak field liquid.

$\mu_{spin} = \sqrt{24}$ BM

$= 4.9$ BM

13.



Which of the following reagent is suitable for the preparation of the product in the above reaction?

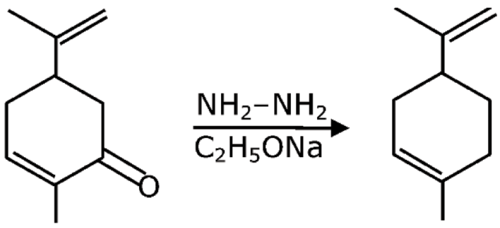
a. Red P + Cl_2

b. NH_2-NH_2/ $C_2H_5O^-Na^+$

c. Ni/H_2

d. $NaBH_4$

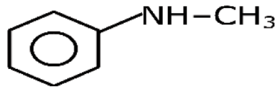
Ans: (b)
Solution:



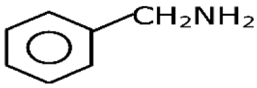
It is wolff-kishner reduction of carbonyl compounds.

14. The diazonium salt of which of the following compounds will form a coloured dye on reaction with β-Naphthol in NaOH?

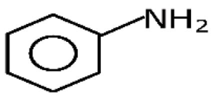
a.



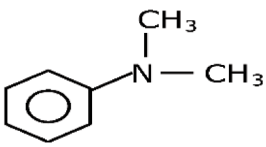
b.



c.

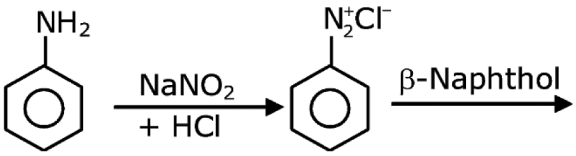


d.



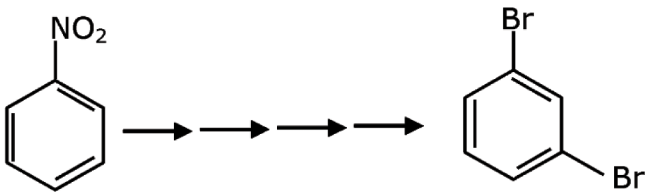
Ans: (c)

Solution:



Orange bright dye.

15. What is the correct sequence of reagents used for converting nitrobenzene into m-dibromobenzene?



a. $\xrightarrow{Sn/HCl}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{NaNO_2}$ / $\xrightarrow{NaBr}$

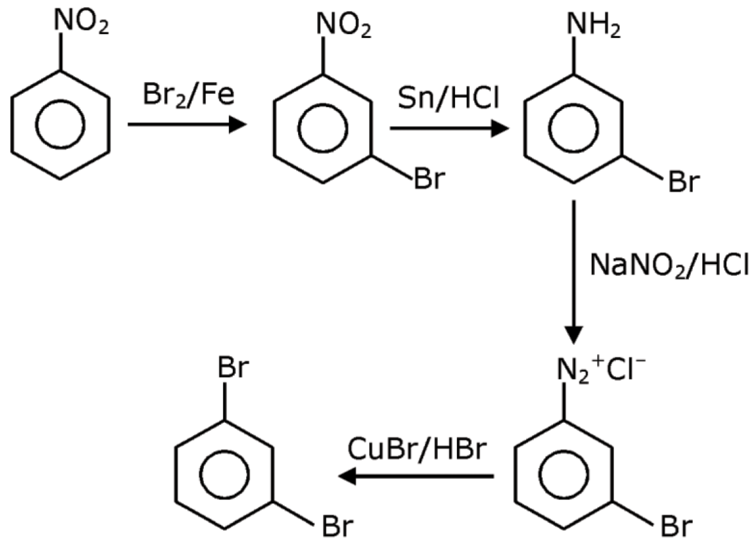
b. $\xrightarrow{Sn/HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{H^+}$

c. $\xrightarrow{NaNO_2}$ / $\xrightarrow{HCl}$ / $\overline{\text{KE}}$

d. $\xrightarrow{Br_2/Fe}$ / $\xrightarrow{Sn/HCl}$ / $\xrightarrow{NaNO_2/HCl}$ / $\xrightarrow{CuBr/HBr}$

Ans: (d)

Solution:

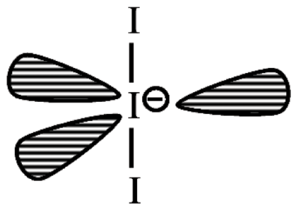


16. The correct shape and I-I-I bond angles respectively in I_3^- ion are:
    a. Trigonal planar; 120°
    b. Distorted trigonal planar; 135° and 90°
    c. Linear; 180º
    d. T-shaped; 180º and 90º

Ans: (c)

Solution:

I_3^- has sp³d hybridization (2 BP + 3 LP) and linear geometry.



17. What is the correct order of the following elements with respect to their density?
    a. Cr < Fe < Co < Cu < Zn
    b. Cr < Zn < Co < Cu < Fe
    c. Zn < Cu < Co < Fe < Cr
    d. Zn < Cr < Fe < Co < Cu

Ans: (d)

Solution:

Fact Based

Density depends on many factors like atomic mass. atomic radius and packing efficiency.

18. Match List-I and List-II.

List – I	List-II
a. $R-\overset{\overset{O}{\parallel}}{C}-Cl \rightarrow R-CHO$	(i) $Br_2/NaOH$
b. $R-CH_2-COOH \rightarrow R-\underset{\underset{Cl}{\mid}}{CH}-COOH$	(ii) $H_2/Pd\text{-}BaSO_4$
c. $R-\overset{\overset{O}{\parallel}}{C}-CH_3 \rightarrow R-CH_2-CH_3$	(iii) Zn (Hg)/Conc. HCl
d. $R-\overset{\overset{O}{\parallel}}{C}-NH_2 \rightarrow R-NH_2$	(iv) Cl_2/Red P, H_2O

a. (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
b. (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
c. (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
d. (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Ans: (d)

Solution:

(a) $R-\overset{\overset{O}{\parallel}}{C}-Cl \xrightarrow{H_2/Pd-BaSO_4} R-CHO$ (Rosenmund reaction)

(b)

$$R-CH_2-COOH \xrightarrow{Cl_2/Red\ P,\ H_2O} R-\underset{\underset{Cl}{\mid}}{CH}-COOH$$

(HVZ reaction)

(c)

$$R-\overset{\overset{\large O}{||}}{C}-NH_2 \xrightarrow{Br_2/NaOH} R-NH_2$$ (Hoffmann Bromamide reaction)

(d)

$$R-\overset{\overset{\large O}{||}}{C}-CH_3 \xrightarrow{Zn(Hg)/conc.HCl} R-CH_2-CH_3$$ (Clemmensen reaction)

19. In polymer Buna-S: 'S' stands for :
   a. Styrene
   b. Sulphur
   c. Strength
   d. Sulphonation

Ans: (a)
Solution:
Buna-S is the co-polymer of buta-1,3-diene & styrene

20. Most suitable salt which can be used for efficient clotting of blood will be:
   a. $Mg(HCO_3)_2$
   b. $FeSO_4$
   c. $NaHCO_3$
   d. $FeCl_3$

Ans: (d)

Solution:
Blood is a negative sol, according to Hardy-Schulz's rule, the cation with high charge has high coagulation power. Hence, $FeCl_3$ can be used for clotting blood.

Section B

1. The magnitude of the change in oxidising power of the MnO4-/ Mn2+ couple is x × 10–4 V, if the H+ concentration is decreased from 1M to 10–4 M at 25°C. (Assume concentration of MnO4- and Mn2+ to be same on change in H+ concentration). The value of x is _____.
(Rounded off to the nearest integer)
$\left[\text{Given : } \frac{2303RT}{F} = 0.059\right]$

Ans: 3776
Solution:
$$5e^- + MnO_4^- + 8H^+ \longrightarrow Mn^{+2} + 4H_2O$$

$$Q = \frac{[Mn^{+2}]}{[H^+]^8[MnO_4^-]} \quad E_1 = E^\circ - \frac{0.059}{5}\log(Q_1)$$

$$E_2 = E^\circ - \frac{0.059}{5}\log(Q_2) \quad E_2 - E_1 = \frac{0.059}{5}\log\left(\frac{Q_1}{Q_2}\right)$$

$$\frac{0.059}{5}\log\left\{\frac{[H^+]_{II}}{[H^+]_I}\right\}^8 \quad \frac{0.059}{5}\log\left(\frac{10^{-4}}{1}\right)^8$$

$$(E_2 - E_1) = \frac{0.059}{5}\times(-32) \quad |(E_2 - E_1)| = 32\times\frac{0.059}{5} = x\times10^{-4}$$

$$\frac{32\times590}{5}\times10^{-4} = x\times10^{-4}$$

$= 3776 \times 10^{-4}$ so, $x = 3776$

2. Among the following allotropic forms of sulphur, the number of allotropic forms, which will show paramagnetism is ______.
   (1) α-sulphur (2) β-sulphur (3) S_2-form

Ans: 1
Solution:
S_2 is like O_2 i.e. paramagnetic as per molecular orbital theory.

3. C_6H_6 freezes at 5.5ºC. The temperature at which a solution of 10 g of C_4H_{10} in 200 g of C_6H_6 freeze is ________ °C. (The molal freezing point depression constant of C_6H_6 is 5.12°C/m).

Ans: 1
Solution:
$\Delta T_f = i \times K_f \times m$

$$= 1 \times 5.12 \times \frac{10/58}{200} \times 1000$$

$$\Delta T_f = \frac{5.12\times50}{58} = 4.414$$

$T_{f(solution)} = T_{K(solvent)} - \Delta T_f = 5.5 - 4.414 = 1.086°C$

$\approx 1.09°C = 1$ (nearest integer)

4. The volume occupied by 4.75 g of acetylene gas at 50°C and 740 mmHg pressure is _______L. (Rounded off to the nearest integer)
   (Given R = 0.0826 L atm K^{-1} mol^{-1})

Ans: 5

Solution:

$T = 50°C = 323.15\ K$

$P = 740\ \text{mm of Hg} = \frac{740}{760}\ atm$

$V = ?$

$\text{moles (n)} = \frac{4.75}{26}\ atm$

$$V = \frac{4.75}{26} \times \frac{0.0821 \times 323.15}{740} \times 760$$

$V = 4.97 \approx 5\ \text{Lit}$

5. The solubility product of PbI_2 is 8.0×10^{-9}. The solubility of lead iodide in 0.1 molar solution of lead nitrate is $x \times 10^{-6}$ mol/L. The value of x is _______ (Rounded off to the nearest integer)
Given $\sqrt{2} = 1.41$

Ans: 141

Solution: $PbI_2(s) \rightleftharpoons Pb^{2+}(aq) + 2I^{-}(aq)$

S+0.1 2s

$K_{SP}(PbI_2) = 8 \times 10^{-9}$

$K_{SP} = [Pb^{+2}][I^{-}]^2$

$8 \times 10^{-9} = (S + 0.1)\ (2S)^2 \quad \Rightarrow (8 \times 10^{-9} + 0.1) \times 4S^2$

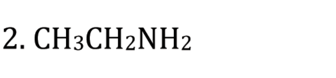
$\Rightarrow S^2 = 2 \times 10^{-8}$

$S = 1.414 \times 10^{-4}$ mol/Lit

$= x \times 10^{-6}$ mol/Lit $\quad \therefore \quad x = 141.4 \approx 141$

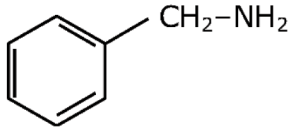
6. The total number of amines among the following which can be synthesized by Gabriel synthesis is ______.

1.

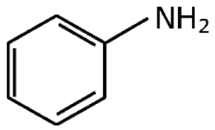


2. $CH_3CH_2NH_2$

3.



4.



Ans: 3

Solution:

Only 1° amines can be prepared by Gabriel synthesis.

7. 1.86 g of aniline completely reacts to form acetanilide. 10% of the product is lost during purification. Amount of acetanilide obtained after purification (in g) is ____× 10–2.

Ans: 243

Solution:

$$Ph-NH_2 \longrightarrow Ph-NH-\overset{\overset{O}{||}}{C}-CH_3$$

(C_6H_7N) (Acetanilide)(C_8H_9NO)

Molar mass = 93 Molar mass = 135

93 g Aniline produce 135 g acetanilide

1.86 g produce $\frac{135\times1.86}{93} = 2.70\ g$

At 10% loss, 90% product will be formed after purification.

∴ Amount of product obtained $= \frac{2.70\times90}{100} = 2.43\ g = 243 \times 10^{-2}$ g

8. The formula of a gaseous hydrocarbon which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 times its own volume of CO_2 is C_xH_y. The value of y is …………

Ans: 8

Solution:

$C_xH_y + 6O_2 \rightarrow 4CO_2 + \frac{y}{2} H_2O$

Applying POAC on 'O' atoms

$6 \times 2 = 4 \times 2 + y/2 \times 1$

$\frac{y}{2} = 4 \Rightarrow y = 8$

9. Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25ºC. After 9h, the fraction of sucrose remaining is f. The value of $\log_{10}\frac{1}{2}$ is ________ $\times 10^{-2}$ (Rounded off to the nearest integer)

[Assume: ln10 = 2.303, ln2 = 0.693]

Ans : 81

Solution:

Sucrose $\xrightarrow{\text{Hydrolysis}}$ Glucose + Fructose

$t_{1/2} = 3.33\text{h} = \frac{10}{3}h$

$C_t = \frac{C_o}{2^{t/t_{1/2}}}$

Fraction of sucrose remaining = f $= \frac{C_t}{C_o} = \frac{1}{2^{\frac{t}{t1}}}$

$\frac{1}{f} = 2^{t/t_{1/2}}$

$\log(1/f) = \log\left(2^{t/t_{1/2}}\right) = \frac{t}{t_{1/2}}\log(2)$

$\frac{9}{10/3} \times 0.3 = \frac{8.1}{10} = 0.81$

$= x \times 10^{-2}$ $\quad x = 81$

10. Assuming ideal behavior, the magnitude of log K for the following reaction at 25ºC is $x \times 10^{-1}$. The value of x is _________.(Integer answer)

$3HC \equiv CH(g) \rightleftharpoons C_6H_6\,(l)$

[Given: $\Delta_f G°(HC \equiv CH) = -2.04 \times 10^5$] mol^{-1}; $\Delta_f G°(C_6H_6) = -1.24 \times 10^5$ J mol^{-1} ;
R = 8.314 J K^{-1} mol^{-1}]

Ans: 855

Solution:

$\Delta G_r^o = \Delta G_f^o\left[C_6H_8(\ell)\right] - 3 \times \Delta G_f^o\left[HC \equiv CH\right]$

$= [-1.24 \times 10^5 - 3x\,(-2.04 \times 10^5)]$

$= 4.88 \times 10^5$ J/mol

$\Delta G_r^o = -RT\ln(K_{eq})$

$\log(K_{eq}) = \frac{-\Delta G^o}{2.303RT}$

$\frac{-4.88 \times 10^5}{2.303 \times 8.314 \times 298} = -8.55 \times 101 = 855 \times 10^{-1}$

1. Consider three observations a, b and c such that b = a+c.If the standard deviation of a+2, b+2, c+2 is d, then which of the following is true?

(1) $b^2 = a^2 + c^2 + 3d^2$

(2) $b^2 = 3\left(a^2 + c^2\right) - 9d^2$

(3) $b^2 = 3\left(a^2 + c^2\right) + 9d^2$

(4) $b^2 = 3\left(a^2 + c^2 + d^2\right)$

Ans. **(2)**

Sol. for a, b, c

mean = $\bar{x} = \frac{a+b+c}{3}$

$\bar{x} = \frac{2\,b}{3}$

S.D. of a, b, c = d

$d^2 = \frac{a^2 + b^2 + c^2}{3} - \frac{4b^2}{9}$

$b^2 = 3a^2 + 3c^2 - 9d^2$

2. Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{\imath} + \hat{\jmath}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and (0, 0) is equal to:

(1) 1

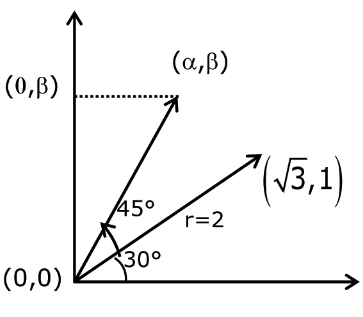
(2) $\frac{1}{2}$

(3) $\frac{1}{\sqrt{2}}$

(4) $2\sqrt{2}$

Ans. **(2)**

Sol.



$(\alpha, \beta) \equiv (2\cos 75°, 2\sin 75°)$

Area = $\frac{1}{2}$ (2 cos75°)(2 sin 75°)

= sin(150°) = $\frac{1}{2}$ square unit

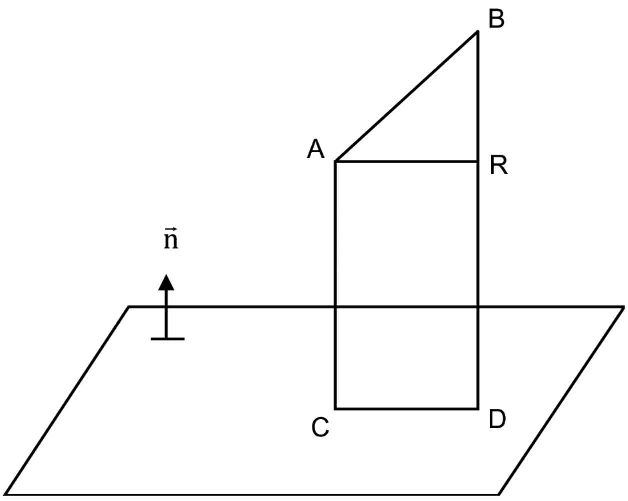
3. If for a>0, the feet of perpendiculars from the points A(a, −2a, 3) and B(0, 4, 5) on the plane $lx + my + nz = 0$ are points C(0, −a, −1) and D respectively, then the length of line segment CD is equal to :

(1) $\sqrt{41}$ (3) $\sqrt{31}$

(2) $\sqrt{55}$ (4) $\sqrt{66}$

Ans. (4)

Sol.



Direction cosines of plane =λ(direction cosines of line AC)

$\therefore$ direction cosines of plane = $\lambda a, -\lambda a, 4\lambda$

Hence equation plane is: $ax - ay + 4z = 0$

$\because$ point C lies on plane

$\therefore a(0) - a(-a) + 4(-1) = 0 \Rightarrow a = 2 \quad (\because a > 0)$

So plane is $2x - 2y + 4z = 0$, $C \equiv (0, -2, -1)$

So for coordinates of D,

$\frac{x-0}{2} = \frac{y-4}{-2} = \frac{z-5}{4} = -\left(\frac{2(0)-2(4)+4(5)}{2^2+2^2+4^2}\right)$

$D \equiv (-1,5,3)$

$\therefore CD = \sqrt{66}$ unit

4. The range of $a \in R$ for which the function

$f(x) = (4a-3)(x + \log_e 5) + 2(a-7) \cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right),\ x \neq 2n\pi, n \in N$ has critical points, is :

(1) $\left[-\frac{4}{3}, 2\right]$

(2) $[1, \infty)$

(3) $(-\infty, -1]$

(4) $(-3, 1)$

Ans. **(1)**

Sol. $f(x) = (4a-3)(x + \ln 5) + 2(a-7)\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \cdot \sin^2\frac{x}{2}\right)$

$f(x) = (4a-3)(x + \log_e 5) + (a-7)\sin x$

$\Rightarrow f'(x) = (4a-3) + (a-7)\cos x = 0$

$\Rightarrow \cos x = \frac{-(4a-3)}{a-7}$

$\Rightarrow\ -1 \leq -\frac{(4a-3)}{a-7} < 1 \qquad (\because -1 \leq \cos x \leq 1)$

$-1 < \frac{4a-3}{a-7} \leq 1$

$\frac{4a-3}{a-7} - 1 \leq 0$ and $\frac{4a-3}{a-7} + 1 > 0$

$\Rightarrow a \in \left[\frac{4}{3}, 7\right)$ and $a \in (-\infty, 2) \cup (7, \infty)$

$\Rightarrow \frac{-4}{3} \leq a < 2$

5. Let the functions f: R→R and g: R →R be defined as :

$f(x) = \begin{cases} x+2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x^3, & x < 1 \\ 3x-2, & x \geq 1 \end{cases}$

Then, the number of points in R where (fog)(x) is NOT differentiable is equal to :

(1) 1

(2) 2

(3) 3

(4) 0

Ans. **(1)**

Sol. $fog(x) = \begin{cases} x^3 + 2, & x < 0 \\ x^6, & 0 \le x < 1 \\ (3x-2)^2, & x \ge 1 \end{cases}$

Clearly fog(x) is discontinuous at x = 0 then non-differentiable at x = 0

Now,

at x = 1

$\text{RHD} = \lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^+} \frac{(3(1+h)-2)^2 - 1}{h} = 6$

$\text{LHD} = \lim_{h \to 0^-} \frac{f(1-h) - f(1)}{-h} = \lim_{h \to 0^-} \frac{(1-h)^6 - 1}{-h} = 6$

Number of points of non-differentiability = 1

6. Let a complex number z, $|z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}} \left(\frac{|z| + 11}{(|z| - 1)^2} \right) \le 2$. Then, the largest value of |z| is equal to ____

(1) 5
(2) 8
(3) 6
(4) 7

Ans. **(4)**

Sol. $\frac{|z| + 11}{(|z| - 1)^2} \ge \frac{1}{2}$

$2|z| + 22 \ge (|z| - 1)^2$

$2|z| + 22 \ge |z|^2 - 2|z| + 1$

$|z|^2 - 4|z| - 21 \le 0$

$(|z| - 7)(|z| + 3) \le 0$

$\Rightarrow |z| \le 7$

$\therefore |z|_{max} = 7$

7. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

(1) $\frac{3}{4}$
(2) $\frac{52}{867}$
(3) $\frac{39}{50}$
(4) $\frac{22}{425}$

Ans. **(3)**

Sol. $P(\overline{S}_{\text{missing}} \mid \text{both found spade}) = \dfrac{P\left(\overline{S_m} \cap BFS\right)}{P(BFS)}$

$$= \frac{\left(1-\frac{13}{52}\right)\times\frac{13}{51}\times\frac{12}{50}}{\left(1-\frac{13}{52}\right)\times\frac{13}{51}\times\frac{12}{50}+\frac{13}{52}\times\frac{12}{51}\times\frac{11}{50}}$$

$$= \frac{39}{50}$$

8. If n is the number of irrational terms in the expansion of $\left(3^{\frac{1}{4}}+5^{\frac{1}{8}}\right)^{60}$, then (n–1) is divisible by :

(1) 8

(2) 26

(3) 7

(4) 30

Ans. (2)

Sol. $T_{r+1} = {}^{60}C_r\left(3^{1/4}\right)^{60-r}\left(5^{1/8}\right)^r$

rational if $\frac{60-r}{4}, \frac{r}{8}$, both are whole numbers, r $\in \{0,1,2,....60\}$

$\frac{60-r}{4} \in W \Rightarrow r \in \{0,4,8,....60\}$

and $\frac{r}{8} \in W \Rightarrow r \in \{0,8,16,...56\}$

$\therefore$ Common terms r $\in \{0,8,16,....56\}$
So 8 terms are rational
Then irrational terms = 61 – 8 = 53 = n
$\therefore$ n – 1 = 52 = 13 × 2^2
factors 1,2,4,13,26,52

9. Let the position vectors of two points P and Q be $3\hat{i}-\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}-4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are (4, –1, 2) and (–2,1,–2) respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is :

(1) $\sqrt{5}$

(2) $\sqrt{171}$

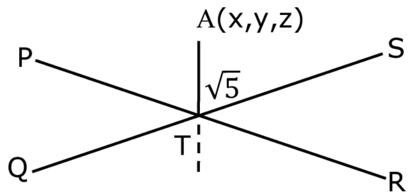
(3) $\sqrt{227}$

(4) $\sqrt{482}$

Ans. **(2)**

Sol. $\vec{p} = 3\hat{i} - \hat{j} + 2\hat{k}$ & $\vec{q} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\overrightarrow{v_{PR}} = (4, -1, 2)$ & $\overrightarrow{v_{QS}} = (-2, 1, -2)$



$L_{PR}: \vec{r} = (3\hat{i} - \hat{j} + 2\hat{k}) + \lambda(4\hat{i} - 1\hat{j} + 2\hat{k})$

$L_{QS}: \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \mu(-2\hat{i} + 1\hat{j} - 2\hat{k})$

Now T on PR = $(3 + 4\lambda, -1 - \lambda, 2 + 2\lambda)$

Similarly T on QS = $(1 - 2\mu, 2 + \mu, -4 - 2\mu)$

For λ & μ: $\left.\begin{array}{l} 3 + 4\lambda = 1 - 2\mu \Rightarrow \mu + 2\lambda = -1 \\ -1 - \lambda = 2 + \mu \Rightarrow \mu + \lambda = -3 \end{array}\right\} \begin{array}{l} \lambda = 2 \\ \mu = -5 \end{array}$

And $2 + 2\lambda = -4 - 2\mu$

$\Rightarrow T: (11, -3, 6)$

D.R. of TA = $\overrightarrow{v_{QS}} \times \overrightarrow{v_{PR}}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -2 \\ 4 & -1 & 2 \end{vmatrix} = 0\hat{i} - 4\hat{j} - 2\hat{k}$

$L_{TA}: \vec{r} = (11\hat{i} - 3\hat{j} + 6\hat{k}) + \lambda(-4\hat{j} - 2\hat{k})$

Now $A = (11, -3 - 4\lambda, 6 - 2\lambda)$

$TA = \sqrt{5}$

$\Rightarrow (4\lambda)^2 + (2\lambda)^2 = 5$

$\Rightarrow 16\lambda^2 + 4\lambda^2 = 5 \Rightarrow \lambda = \pm\frac{1}{2}$

A: (11, –5, 5) or A: (11, –1, 7)

$|A| = \sqrt{121 + 25 + 25}$ or $|A| = \sqrt{121 + 1 + 49}$

$= \sqrt{171}$ or $\sqrt{171}$

10. If the three normals drawn to the parabola, $y^2=2x$ pass through the point (a, 0) $a \neq 0$, then 'a' must be greater than:

(1) 1

(2) $\frac{1}{2}$

(3) $-\frac{1}{2}$

(4) −1

Ans. (1)

Sol. Let the equation of the normal is

$y = mx - 2am - am^3$

here $4a = 2 \Rightarrow a = \frac{1}{2}$

$y = mx - m - \frac{1}{2}m^3$

It passes through A(a, 0) then

$0 = am - m - \frac{1}{2}m^3$

$m = 0,\ m^2 - 2(a-1) = 0$

For real values of m

$2(a - 1) > 0$

$\therefore a > 1$

11. Let $S_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1} + 3^{2r+1}}\right)$.Then $\lim_{k\to\infty} S_k$ is equal to :

(1) $\tan^{-1}\left(\frac{3}{2}\right)$

(2) $\cot^{-1}\left(\frac{3}{2}\right)$

(3) $\frac{\pi}{2}$

(4) $\tan^{-1}(3)$

Ans. (2)

Sol. $\sum_{r=1}^{\infty} \tan^{-1}\left(\frac{6^r(3-2)}{\left(1+\left(\frac{3}{2}\right)^{2r+1}\right)2^{2r+1}}\right)$

$$\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{2^r\cdot 3^{r+1}-3^r 2^{r+1}}{\left(1+\left(\frac{3}{2}\right)^{2r+1}\right)2^{2r+1}}\right)$$

$$\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{\left(\frac{3}{2}\right)^{r+1}-\left(\frac{3}{2}\right)^{r}}{1+\left(\frac{3}{2}\right)^{r+1}\left(\frac{3}{2}\right)^{r}}\right)=\sum_{r=1}^{\infty}\left[\tan^{-1}\left(\frac{3}{2}\right)^{r+1}-\tan^{-1}\left(\frac{3}{2}\right)^{r}\right]=\frac{\pi}{2}-\tan^{-1}\frac{3}{2}=\cot^{-1}\frac{3}{2}$$

12. The number of roots of the equation, $(81)^{\sin^2 x}+(81)^{\cos^2 x}=30$ in the interval [0, π] is equal to :

(1) 3

(3) 4

(2) 2

(4) 8

Ans. **(3)**

Sol. $(81)^{\sin^2 x}+(81)^{1-\sin^2 x}=30$

$(81)^{\sin^2 x}+\frac{81}{(81)^{\sin^2 x}}=30$

Let $(81)^{\sin^2 x}=t$

$t+\frac{81}{t}=30 \quad \Rightarrow t^2+81=30t$

$\Rightarrow t^2-30t+81=0$

$\Rightarrow t^2-27t-3t+81=0$

$\Rightarrow (t-3)(t-27)=0$

$\Rightarrow t=3, 27$

$\Rightarrow (81)^{\sin^2 x}=3, 3^3$

$\Rightarrow 3^{4\sin^2 x}=3^1, 3^3$

$\Rightarrow 4\sin^2 x=1, 3$

$\Rightarrow \sin^2 x=\frac{1}{4}, \frac{3}{4}$

in $[0,\pi]$ $\sin x \geq 0$

$\sin x=\frac{1}{2}, \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}$

Number of solutions = 4

13. If y=y(x) is the solution of the differential equation, $\frac{dy}{dx} + 2y\tan x = \sin x, y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of the function y(x) over **R** is equal to :

(1) 8

(2) $\frac{1}{2}$

(3) $-\frac{15}{4}$

(4) $\frac{1}{8}$

Ans. **(4)**

Sol. $\frac{dy}{dx} + 2\tan x \cdot y = \sin x$

I.F. $= e^{\ln(\sec^2 x)} = \sec^2 x$

$\Rightarrow y\sec^2 x = \int \tan x \sec x \, dx = \sec x + c$

Now $x = \frac{\pi}{3}, y = 0$

$c = -2$

$\therefore y = \cos x - 2\cos^2 x$

$y = -2\left(\cos^2 x - \frac{1}{2}\cos x\right) = -2\left(\left(\cos x - \frac{1}{4}\right)^2 - \frac{1}{16}\right)$

$y = \frac{1}{8} - 2\left(\cos x - \frac{1}{4}\right)^2$

$\therefore \quad y_{max} = \frac{1}{8}$

14. Which of the following Boolean expression is a tautology?

(1) $(p \wedge q) \wedge (p \rightarrow q)$

(2) $(p \wedge q) \vee (p \vee q)$

(3) $(p \wedge q) \vee (p \rightarrow q)$

(4) $(p \wedge q) \rightarrow (p \rightarrow q)$

Ans. **(4)**

Sol.

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T	T
F	T	F	T	T	T
T	F	F	T	F	T
F	F	F	F	T	T

15. Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}, i = \sqrt{-1}$. Then, the system of linear equations $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has :

(1) No solution

(2) Exactly two solutions

(3) A unique solution

(4) Infinitely many solutions

Ans. **(1)**

Sol. $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$A^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}\begin{bmatrix} i & -i \\ -i & i \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = 2\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$A^4 = 4\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = 4\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 8\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^8 = 64\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 64\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$128\begin{bmatrix} x-y \\ -x+y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix} \Rightarrow 128(x-y) = 8$

$\Rightarrow x - y = \frac{1}{16}$ ....(1) and $128(-x+y) = 64 \Rightarrow x - y = \frac{-1}{2}$ ...(2)

$\Rightarrow$ no solution (from eq. (1) & (2))

16. If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, $n > 0$,

then the value of n is equal to :

(1) 16

(2) 20

(3) 12

(4) 9

Ans. **(3)**

Sol. $\log_{10}(\sin x) + \log_{10}(\cos x) = -1$

$\sin x \cdot \cos x = \frac{1}{10}$ ....(1)

and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$

$\Rightarrow \sin x + \cos x = \left(\frac{n}{10}\right)^{\frac{1}{2}}$

$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x \cos x = \frac{n}{10}$ (squaring)

$\Rightarrow 1 + 2\left(\frac{1}{10}\right) = \frac{n}{10}$ (using equation(1))

$\Rightarrow \frac{n}{10} = \frac{12}{10} \Rightarrow n = 12$

17. The locus of the midpoints of the chord of the circle, $x^2+y^2=25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is :

(1) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$

(2) $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$

(3) $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$

(4) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

Ans. **(4)**

Sol. tangent of hyperbola

$y = mx \pm \sqrt{9m^2 - 16}$ ...(i)

which is a chord of circle with mid-point (h, k)
so equation of chord $T = S_1$
$hx + ky = h^2 + k^2$

$y = -\frac{hx}{k} + \frac{h^2 + k^2}{k}$ ...(ii)

by (i) and (ii)

$m = -\frac{h}{k}$ and $\sqrt{9m^2 - 16} = \frac{h^2 + k^2}{k}$

$$9\frac{h^2}{k^2}-16=\frac{\left(h^2+k^2\right)^2}{k^2}$$

locus $9x^2 - 16y^2 = (x^2 + y^2)^2$

18. Let [x] denote greatest integer less than or equal to x. If for $n \in N$,

$\left(1-x+x^3\right)^n=\sum_{j=0}^{3n} a_j x^j$, then $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j}+4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$ is equal to :

(1) 1

(2) n

(3) 2^{n-1}

(4) 2

Ans. **(1)**

Sol. $\left(1-x+x^3\right)^n=\sum_{j=0}^{3n} a_j x^j$

$(1-x + x^3)^n = a_0 + a_1x + a_2x^2 + \ldots.. + a_{3n} x^{3n}$

Put x = 1

$1 = a_0 + a_1 + a_2 + a_3 + a_4 + \ldots\ldots + a_{3n}$ ...(1)

Put x = –1

$1 = a_0 - a_1 + a_2 - a_3 + a_4 \ldots\ldots (-1)^{3n}a_{3n}$ ...(2)

Add (1) + (2)

$\Rightarrow a_0 + a_2 + a_4 + a_6 + \ldots\ldots.. = 1$

Sub (1) – (2)

$\Rightarrow a_1 + a_3 + a_5 + a_7 + \ldots\ldots.. = 0$

Now $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j}+4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$

$= (a_0 + a_2 + a_4 + \ldots..) + 4(a_1 + a_3 + \ldots.)$

$= 1 + 4\times0$

$= 1$

19. Let P be a plane lx+my+nz=0 containing the line, $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$. If plane P divides the line segment AB joining points A(–3, –6, 1) and B(2, 4, –3) in ratio k : 1 then the value of k is equal to :

(1) 1.5

(2) 2

(3) 4

(4) 3

$R : \left(\frac{-3+2k}{k+1}, \frac{-6+4k}{k+1}, \frac{1-3k}{k+1}\right)$ lies on plane

$$8\left(\frac{-3+2k}{k+1}\right)+\left(\frac{-6+4k}{k+1}\right)+2\left(\frac{1-3k}{k+1}\right)=0$$

$-24 + 16k - 6 + 4k + 2 - 6k = 0$

$-28 + 14k = 0$

$k = 2$

20. The number of elements in the set $\{x \in R : (|x| - 3)|x + 4| = 6\}$ is equal to :

(1) 2
(3) 3
(2) 1
(4) 4

Ans. **(1)**

Sol. **Case-1** $x \le -4$

$(-x - 3)(-x - 4) = 6$

$\Rightarrow (x + 3)(x + 4) = 6$

$\Rightarrow x^2 + 7x + 6 = 0$

$\Rightarrow x = -1$ or -6

but $x \le -4$

$x = -6$

Case-2 $x \in (-4, 0)$

$(-x - 3)(x + 4) = 6$

$\Rightarrow -x^2 - 7x - 12 - 6 = 0$

$\Rightarrow x^2 + 7x + 18 = 0$

$D < 0$ No solution

Case-3 $x \ge 0$

$(x - 3)(x + 4) = 6$

$\Rightarrow x^2 + x - 12 - 6 = 0$

$\Rightarrow x^2 + x - 18 = 0$

$$x = \frac{-1 \pm \sqrt{1+72}}{2}$$

$$\therefore x = \frac{\sqrt{73}-1}{2} \text{ only}$$

Hence 2 elements only

Integer Type:

1. Let f: (0, 2) → R be defined as $f(x) = \log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right)$. Then,

$\lim_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+\ldots+f(1)\right)$ is equal to ______

Ans. **(1)**

Sol.

$$E = 2\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$$

$$E = \frac{2}{\ln 2}\int_0^1 \ln\left(1+\tan\frac{\pi x}{4}\right)dx \quad \ldots(i)$$

replacing x →1 − x

$$E = \frac{2}{\ln 2}\int_0^1 \ln\left(1+tan\frac{\pi}{4}(1-x)\right)dx$$

$$E = \frac{2}{\ell n 2}\int_0^1 \ell n\left(1+\tan\left(\frac{\pi}{4}-\frac{\pi}{4}x\right)\right)dx$$

$$E = \frac{2}{\ell n 2}\int_0^1 \ell n\left(1+\frac{1-\tan\frac{\pi}{4}x}{1+\tan\frac{\pi}{4}x}\right)dx$$

$$E = \frac{2}{\ell n 2}\int_0^1 \ell n\left(\frac{2}{1+\tan\frac{\pi x}{4}}\right)dx$$

$$E = \frac{2}{\ell n 2}\int_0^1 \left(\ell n 2-\ell n\left(1+\tan\frac{\pi x}{4}\right)\right)dx \quad \ldots.(ii)$$

equation (i) + (ii)
E = 1

2. The total number of 3×3 matrices A having entries from the set {0, 1, 2, 3} such that the sum of all the diagonal entries of AA^T is 9, is equal to ____

Ans. **(766)**

Sol. $AA^T = \begin{bmatrix} x & y & z \\ a & b & c \\ d & e & f \end{bmatrix}\begin{bmatrix} x & a & d \\ y & b & e \\ z & c & f \end{bmatrix}$

$$= \begin{bmatrix} x^2+y^2+z^2 & ax+by+cz & dx+ey+fz \\ ax+by+cz & a^2+b^2+c^2 & ad+be+cf \\ dx+ey+fz & ad+be+cf & d^2+e^2+f^2 \end{bmatrix}$$

$Tr(AA^T) = x^2 + y^2 + z^2 + a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = 9$

all→1	=1
one 3, rest = 0	$\frac{9!}{8!} = 9$
two 2 , one 1 & rest 0	$\frac{9!}{2!6!} = 63 \times 4 = 252$
one 2 , five 1, rest 0	$\frac{9!}{5!3!} = 63 \times 8 = 504$
Total	= 766

3. Let f:R→R be a continuous function such that f(x)+f(x+1)=2, for all $x \in R$. If $I_1 = \int_0^8 f(x)\,dx$ and $I_2 = \int_{-1}^3 f(x)\,dx$, then the value of $I_1 + 2I_2$ is equal to _____

Ans. **(16)**

Sol. f(x) + f(x + 1) = 2 …..(i)
x→(x + 1)
f(x + 1) + f(x + 2) = 2 …..(ii)
by (i) & (ii)
f(x) − f(x + 2) = 0
f(x + 2) = f(x)
f(x) is periodic with T = 2

$$I_1 = \int_0^{2\times4} f(x)dx = 4\int_0^2 f(x)dx$$

$$I_2 = \int_{-1}^3 f(x)dx = \int_0^4 f(x+1)dx = \int_0^4 (2 - f(x))dx$$

$$I_2 = 8 - 2\int_0^2 f(x)dx$$

$I_1 + 2I_2 = 16$

4. Consider an arithmetic series and a geometric series having four initial terms from the set {11,8,21,16,26,32,4}. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to ____

Ans. **(3)**

Sol. By observation
A.P : 11, 16, 21, 26 ....
G.P : 4, 8, 16, 32 ....
So common terms are 16, 256, 4096

5. If the normal to the curve $y(x)=\int\limits_0^x\left(2t^2-15t+10\right)dt$ at a point (a, b) is parallel to the line x+3y = −5, a>1, then the value of |a+6b| is equal to ____

Ans. **(406)**

Sol. $y'(x) = (2x^2 - 15x + 10)$
at point (a, b) normal is
$3 = (2a^2 - 15a + 10)$
$\Rightarrow 2a^2 - 15a + 7 = 0$
$\Rightarrow 2a^2 - 14a - a + 7 = 0$
$\Rightarrow 2a(a - 7) - 1(a - 7) = 0$

$a = \frac{1}{2}$ or 7,

given a > 1 $\therefore a = 7$
also P lies on curve

$$\therefore b=\int_0^a\left(2t^2-15t+10\right)dt$$

$$b=\int_0^7\left(2t^2-15t+10\right)dt$$

$6b = -413$
$\therefore |a + 6b| = 406$

6. If $\lim\limits_{x\to0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$, then a+b+c is equal to _______

Ans. **(4)**

Sol. $$\lim_{x\to0}\frac{\left\{a\left(1+x+\frac{x^2}{2!}+\ldots\right)-b\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}\ldots\right)+c\left(1-x+\frac{x^2}{2!}\ldots\right)\right\}}{x\left(x-\frac{x^3}{3!}+\ldots\right)}=2$$

$$\therefore\lim_{x\to0}\frac{(a-b+c)+x(a-c)+x^2\left(\frac{a}{2}+\frac{b}{2}+\frac{c}{2}\right)+\ldots.}{x^2\left(1-\frac{x^2}{6}\ldots\right)}=2$$

$\therefore a - b + c = 0$

$\& a - c = 0$

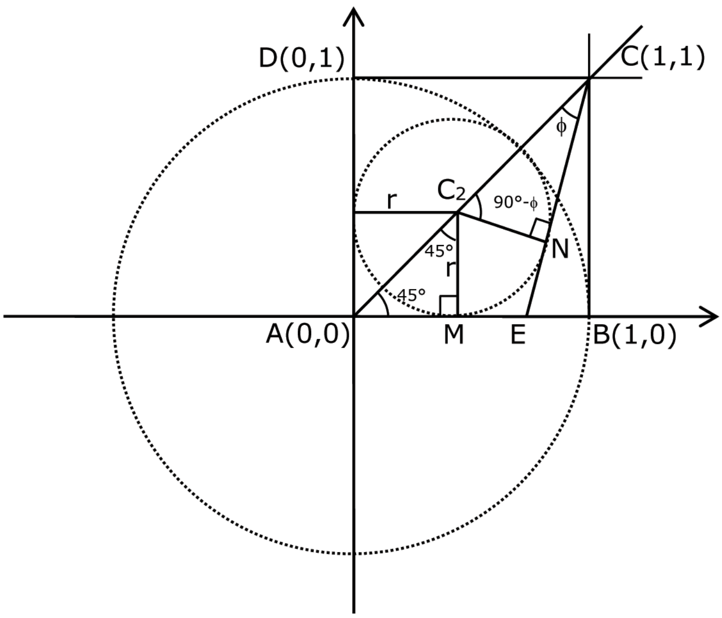
$\& \frac{a}{2} + \frac{b}{2} + \frac{c}{2} = 2$

$\Rightarrow a + b + c = 4$

7. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha + \sqrt{3}\beta$, where α, β are integers, then $\alpha + \beta$ is equal to ___

Ans. **(1)**

Sol.



(i) $\sqrt{2}r + r = 1$

$r = \frac{1}{\sqrt{2}+1}$

$r = \sqrt{2} - 1$

(ii) $CC_2 = 2\sqrt{2} - 2 = 2\left(\sqrt{2}-1\right)$

From ΔCC_2N = $\sin\phi = \frac{\sqrt{2}-1}{2\left(\sqrt{2}-1\right)}$

$\phi = 30°$

(iii) In ΔACE are sine law

$$\frac{AE}{\sin\phi} = \frac{AC}{\sin 105°}$$

$$AE = \frac{1}{2} \times \frac{\sqrt{2}}{\sqrt{3}+1} . 2\sqrt{2}$$

$$AE = \frac{2}{\sqrt{3}+1} = \sqrt{3} - 1$$

$$\therefore EB = 1 - \left(\sqrt{3} - 1\right)$$

$$2 - \sqrt{3}$$

$$\alpha = 2, \beta = -1 \Rightarrow \alpha + \beta = 1$$

8. Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2, \left|\frac{z+i}{z-3i}\right| = 1$and Re (w) has

minimum value. Then, the minimum value of $n \in N$ for Which w^n is real, is equal to ____

Ans. **(4)**

Sol. Let z = x + iy

$|z + i| = |z - 3i|$

$\Rightarrow y = 1$

Now $w = x^2 + y^2 - 2x - 2iy + 2$

$w = x^2 + 1 - 2x - 2i + 2$

$Re(w) = x^2 - 2x + 3$

$Re(w) = (x-1)^2 + 2$

$Re(w)_{min}$ at $x = 1 \Rightarrow z = 1 + i$

Now $w = 1 + 1 - 2 - 2i + 2$

$$w = 2(1-i) = 2\sqrt{2}e^{i\left(\frac{-\pi}{4}\right)}$$

$$w^n = 2\sqrt{2}e^{i\left(\frac{-n\pi}{4}\right)}$$

If w^n is real $\Rightarrow n = 4$

9. Let $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$ where $\omega = \frac{-1+i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $\left(P^{-1}AP - I_3\right)^2$ is $\alpha\omega^2$, then the value of α is equal to _____

Ans. **(36)**

Sol. $\left|P^{-1}AP - I\right|^2$

$= |(P^{-1}AP - I)(P^{-1}AP - I)|$

$= \left|P^{-1}APP^{-1}AP - 2P^{-1}AP + I\right|$

$= \left|P^{-1}A^2P - 2P^{-1}AP + P^{-1}IP\right|$

$= \left|P^{-1}\left(A^2 - 2A + I\right)P\right|$

$= \left|P^{-1}\left(A - I\right)^2 P\right|$

$= \left|P^{-1}\right|\left|A - I\right|^2 \left|P\right|$

$= \left|A - I\right|^2$

$= \begin{vmatrix} 1 & 7 & \omega^2 \\ -1 & -\omega-1 & 1 \\ 0 & -\omega & -\omega \end{vmatrix}^2$

$= \left(1\left(\omega(\omega+1) + \omega\right) - 7\omega + \omega^2.\omega\right)^2$

$= \left(\omega^2 + 2\omega - 7\omega + 1\right)^2$

$= \left(\omega^2 - 5\omega + 1\right)^2$

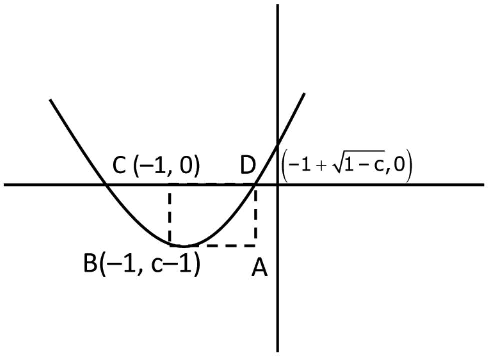
$= \left(-6\omega\right)^2$

$= 36\omega^2 \Rightarrow \alpha = 36$

10. Let the curve y=y(x) be the solution of the differential equation, $\frac{dy}{dx} = 2(x+1)$. If the numerical value of area bounded by the curve y=y(x) and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of y(1) is equal to ___

Ans. **(2)**

Sol. $y = x^2 + 2x + c$



Area of rectangle $(ABCD) = |(c-1)(\sqrt{1-c})|$

Area of parabola and x-axis = $2\left(\frac{2}{3}\left((1-c)^{3/2}\right)\right) = \frac{4\sqrt{8}}{3}$

$1 - c = 2 \Rightarrow c = -1$
Equation of $f(x) = x^2 + 2x - 1$
$f(1) = 1 + 2 - 1 = 2$

1. **A 25 m long antenna is mounted on an antenna tower. The height of the antenna tower is 75 m. The wavelength (in meter) of the signal transmitted by this antenna would be:**

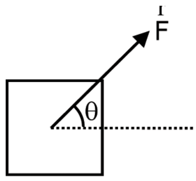
a. 200 b. 400
c. 100 d. 300

Answer: (c)

Sol.

Given that, height of peak of antenna : H = 25 m.
As, we know that
$\lambda = 4H$
$\therefore\ \lambda = 4\times25$
$\therefore\ \lambda = 100$ m
Hence option (c) is correct.

2. **A block of mass m slides along a floor while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic friction is μ_K. Then, the block's acceleration 'a' is given by : (g is acceleration due to gravity)**

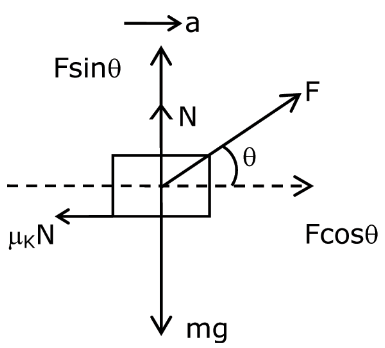


a. $\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$
b. $\frac{F}{m}\cos\theta - \mu_K\left(g + \frac{F}{m}\sin\theta\right)$
c. $\frac{F}{m}\cos\theta + \mu_K\left(g - \frac{F}{m}\sin\theta\right)$
d. $-\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

Answer: (a)

Sol.

Drawing the FBD of the block.



$\Rightarrow$ N = mg – Fsinθ ...(1)

Also, Fcosθ – μ_KN = m·a ...(2)

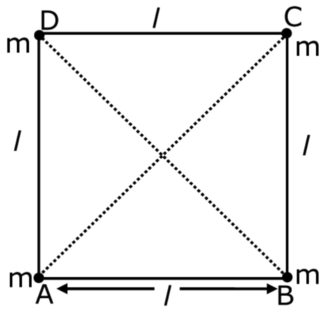
Substituting the value of N from eq. (1) in eq. (2)

$\Rightarrow$ Fcosθ – μ_K(mg–Fsinθ) = m·a

$\Rightarrow a = \frac{F}{m}\cos\theta - \mu_K(g - \frac{F}{m}\sin\theta)$

Hence option (a) is correct.

3. **Four equal masses, m each are placed at the corners of a square of length(*l*) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be :**



a. ml^2

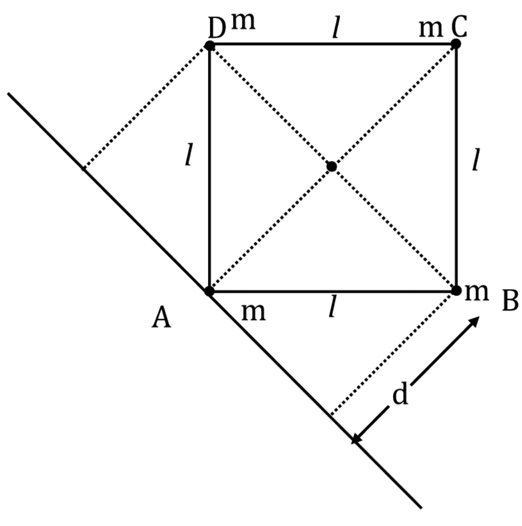
b. $3ml^2$

c. $\sqrt{3}ml^2$

d. $2\ ml^2$

Answer: (b)

Sol.



$AC = \sqrt{l^2 + l^2}$

$AC = l\sqrt{2}$

$d = \frac{l\sqrt{2}}{2}$

$\Rightarrow d = \frac{l}{\sqrt{2}}$

Moment of inertia about the axis passing through A :

$I = m(0)^2 + m(d)^2 + m(d)^2 + m(AC)^2$

$\Rightarrow I = 0 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(l\sqrt{2}\right)^2$

$\Rightarrow I = \frac{ml^2}{2} + \frac{ml^2}{2} + 2ml^2$

$\Rightarrow I = 3ml^2$

Hence option (b) is correct.

4. The stopping potential in the context of photoelectric effect depends on the following property of incident electromagnetic radiation:

a. Amplitude
b. Phase
c. Frequency
d. Intensity

Answer: (c)

Sol.

According to Einstein's photoelectric equation, stopping potential depends on frequency as

$h\nu - h\nu_0 = eV$

$\Rightarrow V = \frac{h}{e}\nu - \frac{h}{e}\nu_0$

Hence stopping potential depends on frequency.

Hence option (c) is correct.

5. One main scale division of a vernier callipers is 'a' cm and n^{th} division of the vernier scale coincide with $(n-1)^{th}$ division of the main scale. The least count of the callipers in mm is:

a. $\left(\frac{n-1}{10n}\right)a$
b. $\frac{10a}{n}$
c. $\frac{10na}{(n-1)}$
d. $\frac{10\,a}{(n-1)}$

Answer: (b)

Sol.

MSD $\rightarrow$ Main scale division

VSD $\rightarrow$ Vernier scale division

LC $\rightarrow$ Least count

n VSD = (n−1) MSD

$1\ VSD = \left(\frac{n-1}{n}\right) MSD$

L·C=1 MSD – 1 VSD

$= 1\,MSD - \left(\frac{n-1}{n}\right) MSD$

$= 1\,MSD - 1\,MSD + \frac{MSD}{n}$

$= \frac{MSD}{n}$

$= \frac{a}{n} cm$

$= \frac{10\,a}{n} mm$

Hence option (b) is correct.

6. A plane electromagnetic wave of frequency 500 MHz is travelling in vacuum along y-direction. At a particular point in space and time, $\vec{B} = 8.0 \times 10^{-8} \hat{z}\, T$. The value of electric field at this point is: (speed of light = 3×10^8 ms^{-1}) Assume $\hat{x}, \hat{y}, \hat{z}$ are unit vectors along x, y and z directions.

a. $2.6\hat{x}\,\frac{V}{m}$

b. $-2.6\hat{y}\,\frac{V}{m}$

c. $24\hat{x}\,\frac{V}{m}$

d. $-24\hat{x}$ V/m

Answer: (d)

Sol.

$E_0 = B \cdot C$

$E_0 = (8 \times 10^{-8}) \times (3 \times 10^{8})$

$\Rightarrow |E_0| = 24$

Wave travels in the direction of $\vec{E} \times \vec{B}$

As $(-\hat{x}) \times \hat{z} = +\hat{y}$

$\therefore\ \hat{E} = -24\hat{x}$ V/m

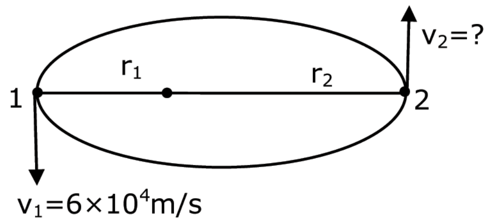
Hence option (d) is correct.

7. The maximum and minimum distances of a comet from the Sun are 1.6×10^{12} m and 8.0×10^{10} m respectively. If the speed of the comet at the nearest point is 6×10^4 ms^{-1}, the speed at the farthest point is :

a. 1.5×10^3 m/s b. 4.5×10^3 m/s
c. 3.0×10^3 m/s d. 6.0×10^3 m/s

Answer: (c)

Sol.



Let point 1 is nearest point,
and point 2 is farthest point.
Given, $r_1 = 8\times10^{10}$ m & $r_2 = 1.6\times10^{12}$ m
By angular momentum conservation
$L_1 = L_2$
$mr_1v_1 = mr_2v_2$
$\Rightarrow v_2 = \frac{r_1 v_1}{r_2}$
$\therefore v_2 = \frac{8\times10^{10}\times6\times10^4}{1.6\times10^{12}}$
$\therefore v_2 = 3.0\times10^3$ m/s
Hence option (c) is correct.

8. A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is:

a. 6.28×10^{-3} N b. 0.0314 N
c. 9.859×10^{-2} N d. 9.859×10^{-4} N

Answer: (d)

Sol. Normal force will provide the necessary centripetal force.
$\Rightarrow N = m\omega^2 R$
Also; $\omega = \frac{2\pi}{-}$

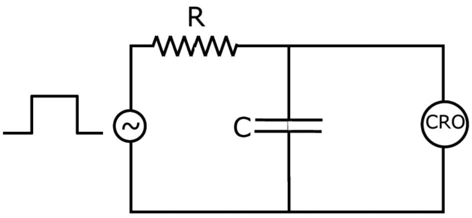
$N = (0.2)\left(\frac{4\pi^2}{T^2}\right)(0.2)$

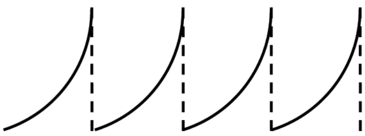
$\Rightarrow N = 0.2 \times \frac{4 \times (3.14)^2}{(40)^2} \times 0.2$

$\therefore$ N=9.859×10^{-4} N

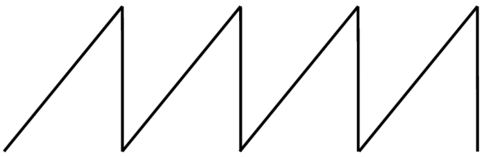
Hence option (d) is correct.

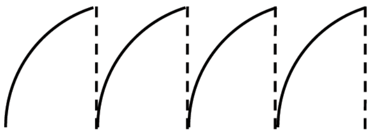
9. An RC circuit as shown in the figure is driven by a AC source generating a square wave. The output wave pattern monitored by CRO would look close to:



a. 

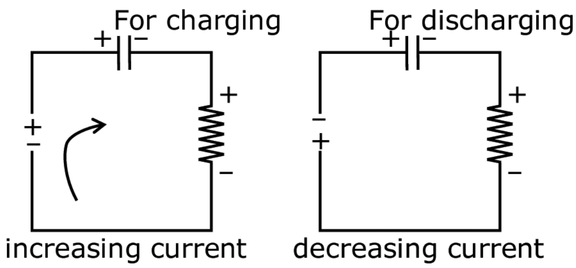
b. 

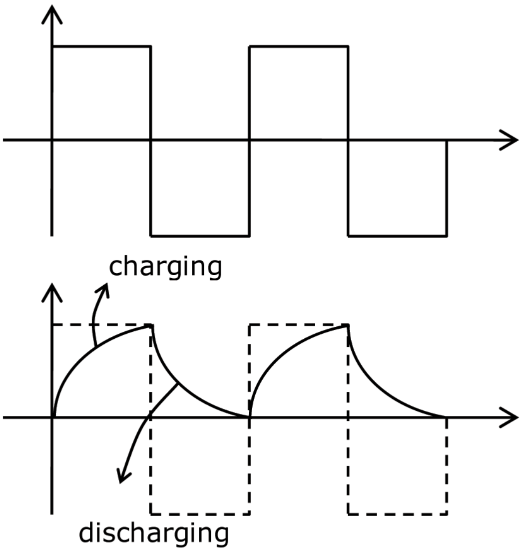
c. 

d. 

Answer: (b)

Sol. Assuming AC starts with positive voltage. When +ve voltage is across input then the capacitor starts charging, trying to reach saturation value, till then there is +ve voltage across input. When –ve voltage of AC appears across input, the capacitor starts discharging till then there is –ve voltage across input and this process of charging and discharging keeps on going alternatively.





Hence option (b) is correct.

10. In thermodynamics, heat and work are:

a. Intensive thermodynamics state variables

b. Extensive thermodynamics state variables

c. Path functions

d. Point functions

Answer: (c)

Sol.

Heat and work are path functions. Heat and work depends on the path taken to reach the final state from initial state. Intensive and extensive properties only applies to physical properties that are a function of state, heat is neither intensive nor extensive. Hence option (c) is correct.

11. A conducting wire of length '*l*', area of cross-section A and electric resistivity ρ is connected between the terminals of a battery. A potential difference V is developed between its ends, causing an electric current. If the length of the wire of the same material is doubled and the area of cross-section is halved, the resultant current would be:

a $\frac{1}{4}\frac{\rho l}{VA}$

b. $\frac{3}{4}\frac{VA}{\rho l}$

c. $4\frac{VA}{\rho l}$

d. $\frac{1}{4}\frac{VA}{\rho l}$

Answer: (d)

Sol.

We know that

$R = \rho \frac{l}{A}$

Now, new length : $l'=2l$

new area of cross section : $A'=A/2$

$\therefore$ New resistance : $R' = \rho \cdot \frac{2l}{A/2}$

$\Rightarrow R' = 4\frac{\rho l}{A}$

$\Rightarrow R'=4R$

$\therefore$ Resultant current : $I = \frac{V}{4R}$

$\therefore I = \frac{1}{4}\frac{VA}{\rho l}$

Hence option (d) is correct.

12. The pressure acting on a submarine is 3×10^5 Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acting on the submarine would be :(Assume that atmospheric pressure is 1×10^5 Pa, density of water is 10^3 kg m^{-3}, acceleration due to gravity g = 10 ms^{-2})

a. $\frac{200}{3}\%$

b. $\frac{5}{200}\%$

c. $\frac{200}{5}\%$

d. $\frac{3}{200}\%$

Answer: (a)

Sol.

Pressure at depth h is

$P = P_0 + h\rho g = 3\times10^5$ Pa

$\Rightarrow h\rho g = 3\times10^5 - 1\times10^5$

$\Rightarrow h\rho g = 2\times10^5$

As h is doubled

$\therefore\ 2h\rho g = 4\times10^5$

$\therefore$ Increased pressure, $P'=P_0+4\times10^5$

$\therefore\ P'=5\times10^5$Pa

$\therefore$% increase in pressure $= \frac{P'-P}{P} \times 100$

$= \frac{(5-3)\times10^5}{3\times10^5} \times 100$

$= \frac{200}{3}\%$

Hence option (a) is correct.

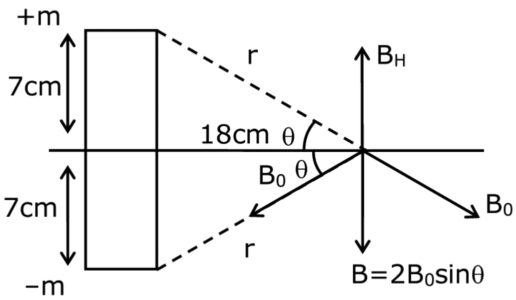
13. A bar magnet of length 14 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north pole. A neutral point is obtained at a distance of 18 cm from the center of the magnet. If B_H = 0.4 G, the magnetic moment of the magnet is (1 G = 10^{-4}T)

a. 28.80 J T^{-1} b. 2.880 J T^{-1}

c. 2.880×10^3 J T^{-1} d. 2.880×10^2 J T^{-1}

Answer: (b)

Sol.



M → magnetic moment of the magnet

m → **power of magnetic pole**

θ → angle made by B_0 with the horizontal

B_0 → **magnetic flux density**

$B=2B_0\sin\theta$

$$B = 2\frac{\mu_0}{4\pi}\frac{m}{r^2}\times\frac{7}{r}$$

$$\Rightarrow 0.4\times10^{-4} = 2\times10^{-7}\times\frac{m\times7}{(7^2+18^2)^{3/2}}\times10^4$$

$$\therefore\ m = \frac{4\times10^{-2}\times(373)^{3/2}}{14}$$

$$\therefore\ M = m\times14\,cm = m\times\frac{14}{100}$$

$$\therefore\ M = \frac{0.04\times(373)^{3/2}}{14}\times\frac{14}{100}$$

∴ M=2.880 J/T

Hence option (b) is correct.

14. The volume V of an enclosure contains a mixture of three gases, 16 g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature T. Consider R as universal gas constant. The pressure of the mixture of gases is:

a. $\frac{4RT}{V}$ b. $\frac{88RT}{V}$

c. $\frac{5}{2}\frac{RT}{V}$ d. $\frac{3RT}{V}$

Answer: (c)

Sol.

No. of moles of O_2 : $n_1=\frac{16}{32}=0.5$ mole

No. of moles of N_2 : $n_2=\frac{28}{28}=1$ mole

No. of moles of CO_2 : $n_3=\frac{44}{44}=1$ mole

Total no. of moles in container : $n=n_1+n_2+n_3$

$\therefore n=0.5+1+1=\frac{5}{2}$moles

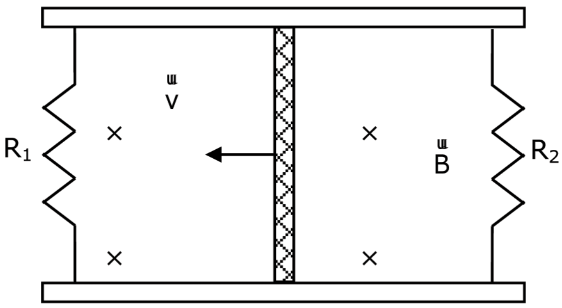
Now; PV=nRT

$P=\frac{nRT}{V}$

$\therefore P=\frac{5}{2}\frac{RT}{V}$

Hence option (c) is correct.

15. A conducting bar of length L is free to slide on two parallel conducting rails as shown in the figure



Two resistors R_1 and R_2 are connected across the ends of the rails. There is a uniform magnetic field $\vec{B}$ pointing into the page. An external agent pulls the bar to the left at a constant speed *v.* The correct statement about the directions of induced currents I_1 and I_2 flowing through R_1 and R_2 respectively is :

a. I_1 is in clockwise direction and I_2 is in anticlockwise direction

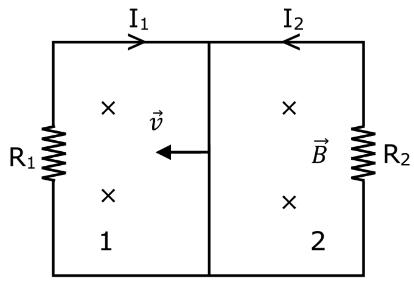
b. Both I_1 and I_2 are in clockwise direction

c. I_1 is in anticlockwise direction and I_2 is in clockwise direction

d. Both I_1 and I_2 are in anticlockwise direction

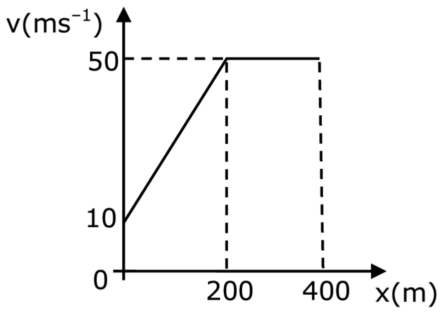
Answer: (a)

Sol.

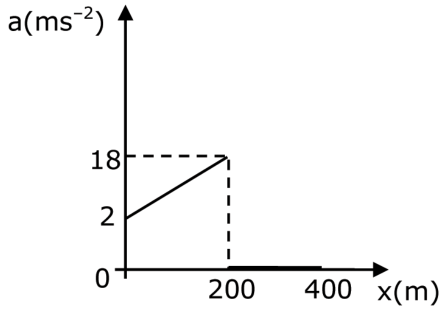


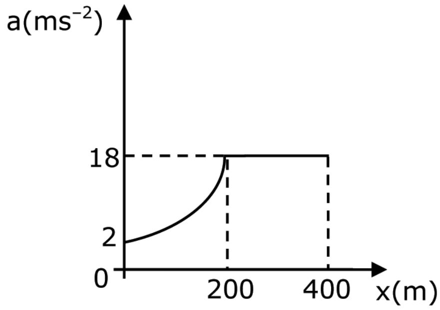
When bar slides, area of loop 1 decreases and that of loop 2 increases. Magnetic flux decreases in 1 and increases in 2. Therefore induced emf and current resist this change. As a result B should increase in 1 and decrease in 2. So I_1 should be clockwise and I_2 anticlockwise.
Hence option (a) is correct.

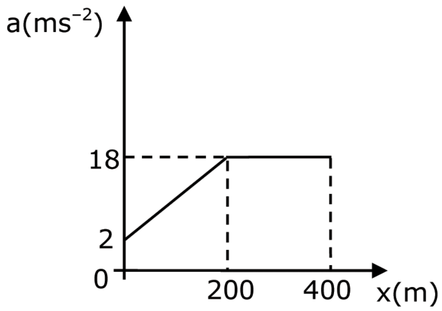
16. **The velocity-displacement graph describing the motion of a bicycle is shown in the figure.**

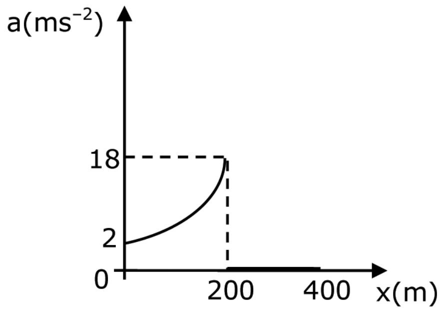


The acceleration-displacement graph of the bicycle's motion is best described by :

a.


b.


c.


d.


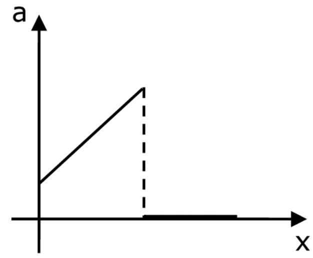
Answer: (a)

Sol.

We know that, $a = v\frac{dv}{dx}$

As slope is constant, so a $\propto$ v (from x=0 to 200 m) & Also, slope = 0, so a = 0 (from x=200 to 400 m)

Hence, the correct plot is

$\therefore$ 

Hence option (a) is correct.

17. For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab is $\frac{3}{4}d$, where 'd' is the separation between the plates of parallel plate capacitor. The new capacitance (C') in terms of original capacitance (C_0) is given by the following relation:

a. $C' = \frac{4K}{K+3}C_0$

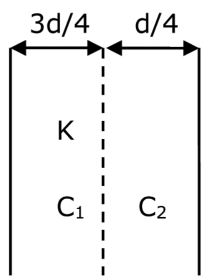
b. $C' = \frac{4}{3+K}C_0$

c. $C' = \frac{3+K}{4K}C_0$

d. $C' = \frac{4+K}{3}C_0$

Answer: (a)

Sol.



$$C_0 = \frac{\epsilon_0 A}{d}$$

C_1 and C_2 are in series and C' is new capacitance

$$\therefore \frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C'} = \frac{(3d/4)}{\epsilon_0\, KA} + \frac{(d/4)}{\epsilon_0\, A}$$
$$\frac{1}{C'} = \frac{d}{4\,\epsilon_0\, A}\left(\frac{3+K}{K}\right)$$
$$\therefore C' = \frac{4K}{(K+3)}C_0$$
Hence option (a) is correct.

18. For an electromagnetic wave travelling in free space, the relation between average energy densities due to electric (U_e) and magnetic (U_m) fields is :

a. $U_e \neq U_m$
b. $U_e = U_m$
c. $U_e > U_m$
d. $U_e < U_m$

Answer: (b)

Sol.

In EMW, average energy density due to electric field (U_e) and magnetic field (U_m) is same.
Hence option (b) is correct.

19. Time period of a simple pendulum is T inside a lift when the lift is stationary. If the lift moves upwards with an acceleration g/2, the time period of pendulum will be:

a. $\sqrt{\frac{3}{2}}T$
b. $\frac{T}{\sqrt{3}}$
c. $\sqrt{\frac{2}{3}}T$
d. $\sqrt{3}T$

Answer: (c)

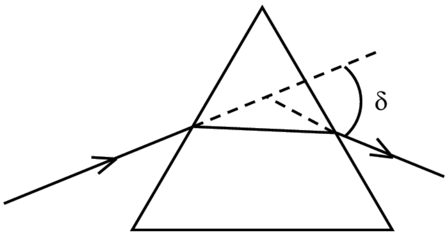
Sol.

When lift is stationary
$$T = 2\pi\sqrt{\frac{L}{g}}$$
A pseudo force will act downwards when lift is accelerating upwards.
$$\therefore\; g_{eff} = g + \frac{g}{2} = \frac{3g}{2}$$
$\therefore$ New time period
$$T' = 2\pi\sqrt{\frac{L}{g_{eff}}}$$
$$T' = 2\pi\sqrt{\frac{2L}{3g}}$$

$\therefore \boxed{T' = \sqrt{\frac{2}{3}}T}$

Hence option (c) is correct.

20. The angle of deviation through a prism is minimum when



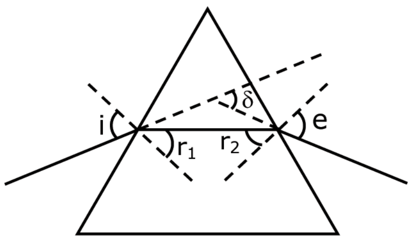
(A) Incident ray and emergent ray are symmetric to the prism
(B) The refracted ray inside the prism becomes parallel to its base
(C) Angle of incidence is equal to that of the angle of emergence
(D) When angle of emergence is double the angle of incidence

Choose the correct answer from the options given below:

a. Only statement (D) is true
b. Statements (A), (B) and (C) are true
c. Statements (B) and (C) are true
d. Only statement (A) and (B) are true

Answer: (b)

Sol.



Deviation is minimum in prism when, $i = e$, $r_1 = r_2$ and ray inside prism is parallel to base of prism.

Hence option (b) is correct.

1. **A fringe width of 6 mm was produced for two slits separated by 1 mm apart. The screen is placed 10 m away. The wavelength of light used is 'x' nm. The value of 'x' to the nearest integer is ______.**

Answer: (600)

Sol.

β = 6 mm, d =1 mm, D =10 m

λ = ?

$$\beta = \frac{\lambda D}{d}$$

$$6 \times 10^{-3} = \frac{\lambda \times 10}{1 \times 10^{-3}}$$

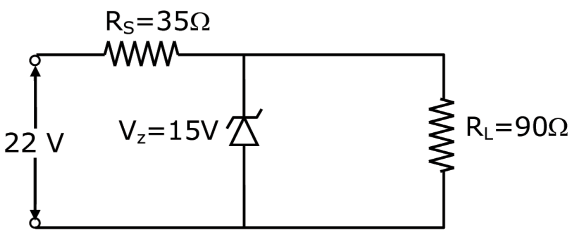
$$\therefore \lambda = \frac{6 \times 10^{-3} \times 1 \times 10^{-3}}{10}$$

$\lambda = 600 \times 10^{-9}$ m

$\therefore \lambda$ = 600 nm

$\therefore$ 600 is the required value.

2. **The value of power dissipated across the zener diode (V_z = 15 V) connected in the circuit as shown in the figure is x*10^{-1} watt.**

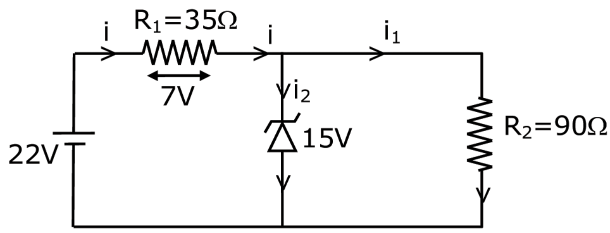


The value of x, to the nearest integer, is _______.

Answer: (5)

Sol.

Across R_1 potential difference is 22 V – 15 V = 7 V



$$i = \frac{7}{35} = \frac{1}{5}A$$

$$i_1 = \frac{15}{90} = \frac{1}{6}A$$

$i_2 = i - i_1$

$i_2 = \frac{1}{5} - \frac{1}{6}$

$i_2 = \frac{1}{30} A$

Power across diode ; P = V_2 i_2

$P = 15 \times \frac{1}{30}$

P = 0.5 W

∴ P = 5×10^{-1} W

∴ 5 is the required value.

3. The resistance $R = \frac{V}{I}$, where V = (50±2) V and I = (20±0.2) A. The percentage error in R is 'x' %. The value of 'x' to the nearest integer is ________.

Answer: (5)

Sol.

$R = \frac{V}{I}$

$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$

% error in $R = \frac{2}{50} \times 100 + \frac{0.2}{20} \times 100$

% error in R = 4+1

∴ % error in R = 5%

∴ 5 is the required value.

4. A sinusoidal voltage of peak value 250 V is applied to a series LCR circuit, in which R = 8Ω, L=24 mH and C=60 μF. The value of power dissipated at resonant conditions is 'x' kW. The value of x to the nearest integer is ________.

Answer: (4)

Sol.

At resonance, power (P)

$P = \frac{(V_{rms})^2}{R}$

$\therefore P = \frac{\left(250/\sqrt{2}\right)^2}{8}$

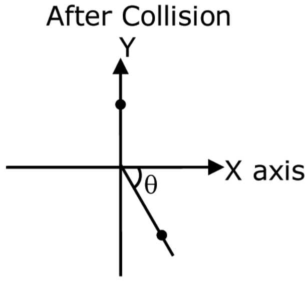
∴ P = 3906.25 W

∴ P ≅ 4 kW

∴ 4 is the required value.

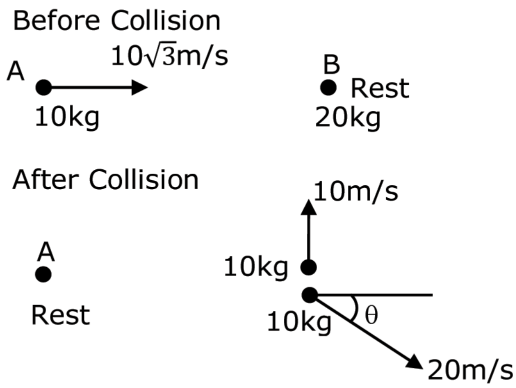
5. A ball of mass 10 kg moving with a velocity $10\sqrt{3}$ ms^{-1} along X-axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the

pieces starts moving along Y-axis at a speed of 10 m/s. The second piece starts moving at a speed of 20 m/s at an angle θ (degree) with respect to the X-axis. The configuration of pieces after collision is shown in the figure. The value of θ to the nearest integer is _______.



Answer: (30)

Sol.



Conserving momentum along x-axis

$\vec{p_i} = \vec{p_f}$

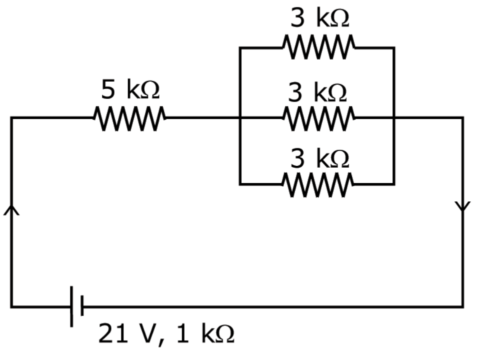
$10 \times 10\sqrt{3} = 10 \times 20\cos\theta$

$\cos\theta = \frac{\sqrt{3}}{2}$

∴ θ = 30°

∴ 30 is the required value.

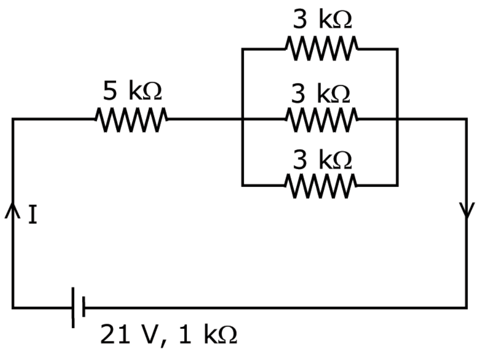
6. **In the figure given, the electric current flowing through the 5 kΩ resistor is 'x' mA.**

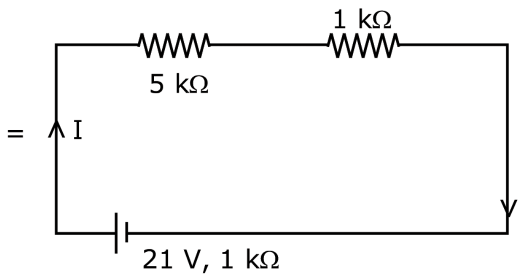


The value of x to the nearest integer is ________.

Answer: (3)

Sol.



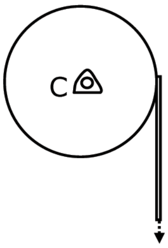


$$I = \frac{21}{5+1+1}$$

$\therefore$ I = 3 mA

$\therefore$ 3 is the required value.

7. Consider a 20 kg uniform circular disk of radius 0.2 m. It is pin supported at its center and is at rest initially. The disk is acted upon by a constant force F=20 N through a massless string wrapped around its periphery as shown in the figure



Suppose the disk makes n number of revolutions to attain an angular speed of 50 rad s^{-1}. The value of n, to the nearest integer is _______. [Given : In one complete revolution, the disk rotates by 6.28 rad]

Answer: (20)

Sol.

$\alpha = \frac{\tau}{I} = \frac{F.R.}{mR^2/2} = \frac{2F}{mR}$

$\alpha = \frac{2 \times 20}{20 \times (0.2)} = 10 \text{ rad/s}^2$

$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$(50)^2 = 0^2 + 2(10)\,\Delta\theta$

$\Rightarrow \Delta\theta = \frac{2500}{20}$

$\Delta\theta = 125$ rad

No. of revolution $= \frac{125}{2\pi} \approx 20$ revolutions.

$\therefore$ 20 is the required value.

8. The first three spectral lines of H-atom in the Balmer series are given λ_1, λ_2, λ_3 considering the Bohr atomic model, the wave lengths of first and third spectral lines $\left(\frac{\lambda_1}{\lambda_3}\right)$ are related by a factor of approximately 'x'$\times 10^{-1}$. The value of x, to the nearest integer, is _______.

Answer: (15)

Sol.

For 1st line

$\frac{1}{\lambda_1} = Rz^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$\frac{1}{\lambda_1} = Rz^2\frac{5}{36}$ ...(i)

For 3rd line

$$\frac{1}{\lambda_3} = Rz^2\left(\frac{1}{2^2} - \frac{1}{5^2}\right)$$

$\frac{1}{\lambda_3} = Rz^2\,\frac{21}{100}$ ...(ii)

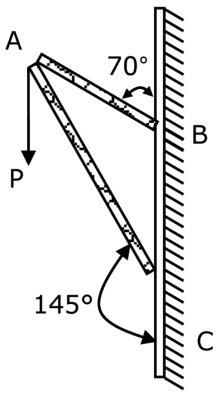
$\therefore\ \frac{(ii)}{(i)}$

$$\frac{\lambda_1}{\lambda_3} = \frac{21}{100} \times \frac{36}{5} = 1.512 = 15.12 \times 10^{-1}$$

$x \approx 15$

$\therefore$ 15 is the required value.

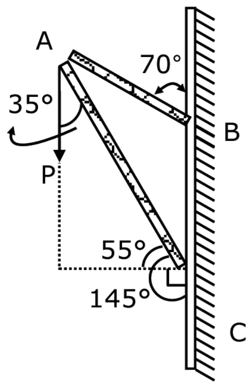
9. **Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force $\vec{P}$ of magnitude 100 N is applied at point A of the frame.**



Suppose the force is $\vec{P}$ resolved parallel to the arms AB and AC of the frame. The magnitude of the resolved component along the arm AC is x N. The value of x, to the nearest integer, is _______. [Given: sin(35°)=0.573, cos(35°)=0.819, sin(110°)=0.939, cos(110°)=–0.342]

Answer: (82)

Sol.



Component along AC

= 100 cos 35°N

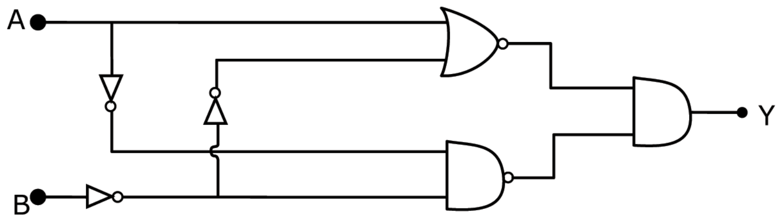
= 100 × 0.819 N

= 81.9 N

≈ 82 N

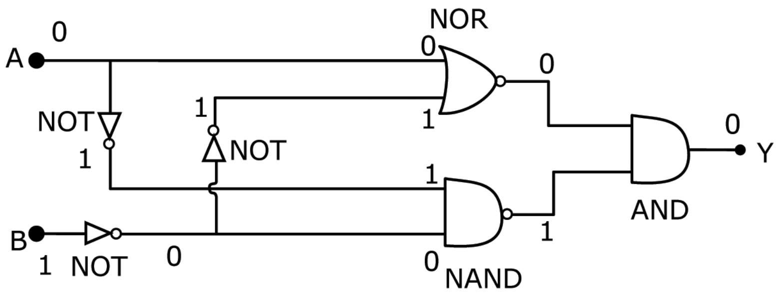
∴ 82 is the required value.

10. In the logic circuit shown in the figure, if input A and B are 0 to 1 respectively, the output at Y would be 'x'. The value of x is _______.



Answer: (0)

Sol.



∴ 0 is the required value.