Memory Based Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2025 (Online) Phase-1

(Physics, Chemistry and Mathematics)

24 JANUARY 2025 (Morning Shift)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains **Three Parts**. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is **Mathematics**. Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt all questions.

(6) **Section - A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section - B (21 – 25) contains** 5 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

1. A drop of radius R is split into 27 drops of equal radius, the work done is 10 J. If the same big drop is split into 64 equal drops the work done is-

(1) 10 J (2) 15 J (3) 20 J (4) $\frac{75}{4}$J

Ans. **(2)**

Sol. $27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$r = \frac{R}{3}$

initial surface area = $4\pi R^2$

Final surface area = $4\pi r^2 \times 27$

Change in surface area = $4\pi\,(27r^2 - R^2)$

$$= 4\pi\left(27\left(\frac{R}{3}\right)^2 - R^2\right) = 8\pi R^2$$

Work done by External Agent = $T \times 8\pi R^2 = 10$ ...... (i)

$64 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow r = \frac{R}{4}$

change in surface area = $4\pi(64r^2 - R^2)$

$$= 4\pi\left(64\left(\frac{R}{4}\right)^2 - R^2\right)$$

$= 12\pi R^2$

Work done by External Agent = $T \times 12\,\pi R^2 = 12 \times \frac{10}{8} = 15J$

2. A satellite is revolving in a stable circular orbit of radius R and time period is T. If orbital radius of an another satellite is 1.03 R, then the percentage charge in time period of the second satellite as compared to the first will be :

(1) 1.5% (2) 4.5% (3) 7.5% (4) 9%

Ans. **(2)**

Sol. $T^2 \propto n^3$

$T \propto n^{3/2}$

$\frac{dT}{T} = \frac{3}{2}\frac{dr}{r} = \frac{3}{2} \times 3\% = 4.5\%$

3. In a parallel plate capacitor, the length, width and separation between the plates are respectively $\ell = 5$ cm, b = 3 cm and d = 1 μm. What will be the dimensions of an another capacitor, so that its capacitance becomes 10 times

(1) ℓ = 50 cm, b = 30 cm, d = 10 μm (2) ℓ = 50 cm, b = 10 cm, d = 10 μm

(3) ℓ = 10 cm, b = 30 cm, d = 50 μm (4) ℓ = 40 cm, b = 10 cm, d = 50 μm

Ans. **(1)**

Sol. $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{\varepsilon_0(\ell b)}{d}$

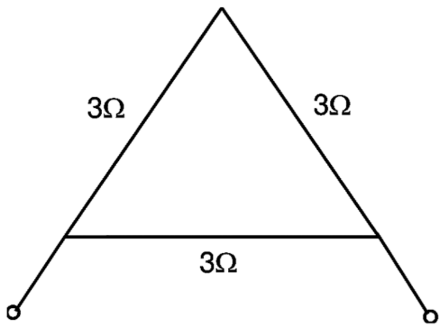
In option (A) $\ell \to$ 10 times, b $\to$ 10 times, b $\to$ 10 so C will also be 10 times.

4. Resistance of uniform wire is 9Ω. If it is bent in the form of an equilateral triangle, then equivalent resistance between its two vertices will be :

(1) 1Ω (2) 2Ω (3) 3 Ω (4) 6Ω

Ans. **(2)**

Sol.



$\dfrac{1}{R_{eq}} = \dfrac{1}{3} + \dfrac{1}{6}$

$R_{eq} = 2\Omega$

5. A Solid cylinder of mass m and radius r is released from rest at the top of a rough inclined plane making an angle of 45° with the horizontal. Assuming the cylinder rolls without sipping find the acceleration of the axis of the cylinder

(1) $\dfrac{g}{2}$ (2) $\dfrac{g}{\sqrt{2}}$ (3) $\dfrac{2g}{3\sqrt{2}}$ (4) $\dfrac{g}{3\sqrt{2}}$

Ans. **(3)**

Sol. $a = \dfrac{g\sin\theta}{1+K^2/R^2}$

$\dfrac{K^2}{R^2} = \dfrac{1}{2}$

$\text{Sin}45° = \dfrac{1}{\sqrt{2}}$

$a = \dfrac{g \times \frac{1}{\sqrt{2}}}{3/2} = \dfrac{2g}{3\sqrt{2}}$

6. If I = I_A Sin ωt + I_B Cos ωt , Then find rms value of current.

(1) $I_{rms} = I_A + I_B$ (2) $I_{rms} = \sqrt{I_A^{\,2} + I_B^{\,2}}$ (3) $I_{rms} = \dfrac{1}{2}\sqrt{I_A^{\,2} + I_B^{\,2}}$ (4) $I_{rms} = \sqrt{\dfrac{I_A^{\,2} + I_B^{\,2}}{2}}$

Ans. **(4)**

Sol. $I_{max} = \sqrt{I_A^2 + I_B^2}$

$I_{rms} = \sqrt{\frac{I_A^2 + I_B^2}{2}}$

7. Work done by a force F = ($\alpha + \beta x^2$) from x = 0 to x = 1 is 5J. If α = 1, then find value of β.

(1) 4 (2) 8 (3) 12 (4) 16

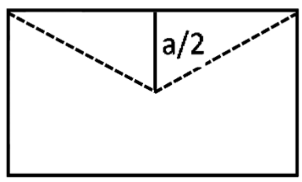
Ans. **(3)**

Sol. $W = \int F dx$

$5 = \alpha x + \beta \frac{x^3}{3}\Big|_0^1$

$5 = 1 \times 1 + \frac{\beta}{3} \times 1 \Rightarrow \beta = 12$

8. In a square loop of side length $\frac{1}{\sqrt{2}}$m, current of 5 Amp is flowing. Find magnetic field as its centre (in μT).



(1) 3 (2) 9 (3) 11 (4) 8

Ans. **(4)**

Sol. $B_C = \frac{\mu_0 i}{\pi l} 2\sqrt{2}$

$= \frac{\mu_0 \times 5}{\pi \frac{1}{\sqrt{2}}} \times 2\sqrt{2}$

$= \frac{\mu_0}{\pi} \times 20 = \frac{4\pi \times 10^{-7} \times 10}{\pi}$

80×10^{-7}

$8 \times 10^{-7} = N \times 10^{-7}$

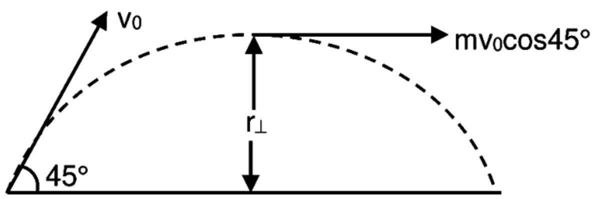
N = 8

9. A body projected with initial velocity V_0 at 45° angle in X-Y plane. Angular momentum of the particle at highest point about point of projection is :

(1) $\frac{mV_0^3}{4g}$ (2) $\frac{mV_0^3}{4\sqrt{2}g}$ (3) $\frac{mV_0^2}{4\sqrt{2}g}$ (4) $\frac{mV_0}{2\sqrt{2}g}$

Ans. **(2)**

Sol.



$\vec{L} = \vec{r} \times \vec{p}$

$\vec{L} = mv_0\cos45°$

$L = mvH$

$= m \times v_0\cos45° \times \frac{u^2 \sin^2 45°}{2g}$

$= mv_0 \times \frac{1}{\sqrt{2}} \times \frac{v_0^2}{4g}$

$L = \frac{mv_0^3}{4\sqrt{2}g} \Rightarrow \vec{L} = \frac{mv_0^3}{4\sqrt{2}g}(-\hat{k})$

10. An electron jumps from principle quantum state A to C by releasing photon of wavelength 2000 Å and from state B to C by releasing of photon of wavelength 6000 Å, then find wavelength of photon for transition from A to B.

(1) 2000 Å (2) 3000 Å (3) 4000 Å (4) 8000 Å

Ans. **(2)**

Sol. $E_A - E_C = \frac{hc}{2000}$

$E_B - E_C = \frac{hc}{6000}$

$E_A - E_B = \frac{hc}{\lambda}$

$E_A - E_B = \frac{hc}{2000} - \frac{hc}{6000} = \frac{hc}{\lambda}$

$\Rightarrow \frac{3-1}{6000} = \frac{1}{\lambda}$

$\lambda = \frac{6000}{2} = 3000Å$

11. Initial velocity of an electron is $v_0\hat{i}$ and its initial De-Broglie wavelength is λ_0. A uniform electric field of $\vec{E} = -E_0\hat{k}$ is applied. The De-Broglie wavelength as a function of time will be :-

(1) $\lambda(t) = \lambda_0$

(2) $\lambda(t) = \dfrac{h}{\sqrt{\left(\frac{1}{\lambda_0}\right)^2 + \left(\frac{eE_0t}{h}\right)^2}}$

(3) $\lambda(t) = \dfrac{1}{\sqrt{\left(\frac{1}{\lambda_0}\right)^2 + \left(\frac{eE_0t}{h}\right)^2}}$

(4) $\lambda(t) = \dfrac{1}{\sqrt{\left(\frac{1}{\lambda_0}\right)^2 + \left(\frac{eE_0t}{mh}\right)^2}}$

Ans. **(3)**

Sol. $\lambda_0 = \dfrac{h}{mV_0} \Rightarrow mV_0 = \dfrac{h}{\lambda_0}$

$\vec{V} = \vec{u} + \vec{a}t \Rightarrow \vec{V}(t) = v_0\hat{i} + \dfrac{(-e)(-E_0\hat{k})}{m}t$

$\vec{V} = v_0\hat{i} + \dfrac{eE_0}{m}t\hat{k} \Rightarrow |V| = \sqrt{v_0^2 + \left(\frac{eE_0}{m}t\right)^2}$

$\lambda_{ab}(t) = \dfrac{h}{m|v|} = \dfrac{h}{m\sqrt{v_0^2 + \left(\frac{eE_0}{m}t\right)^2}} = \dfrac{h}{\sqrt{\left(\frac{h}{\lambda_0}\right)^2 + \left(\frac{eE_0}{m}t\right)^2}}$

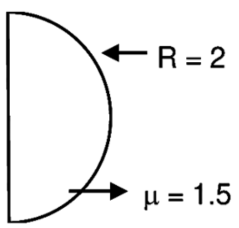
$\lambda_{ab}(t) = \dfrac{1}{\sqrt{\left(\frac{1}{\lambda_0}\right)^2 + \left(\frac{eE_0t}{h}\right)^2}}$

12. Radius of curvature of a plano convex lens is 2 cm and refractive index is 1.5 has focal length f_1 in air and f_2 in a medium of refractive index 1.2, calculate f_1/f_2

(1) 2 : 1 (2) 1 : 2 (3) 3 : 2 (4) 1 : 4

Ans. **(2)**

Sol.



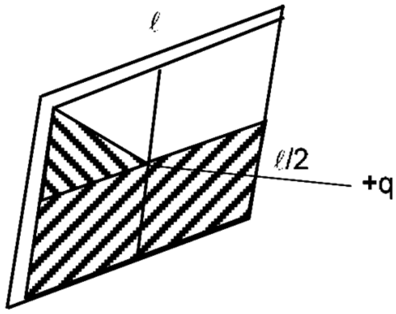
$\dfrac{1}{f_1} = (1.5 - 1) \times \dfrac{1}{2} = \dfrac{1}{4} \Rightarrow f_1 = 4$

$\dfrac{1}{f_2} = \left(\dfrac{1.5}{1.2} - 1\right) \times \dfrac{1}{2} = \dfrac{0.3}{2 \times 1.2} = \dfrac{1}{8}$

$f_2 = 8$

$\dfrac{f_1}{f_2} = \dfrac{4}{8} = \dfrac{1}{2}$

13. A point charge +1C is placed at a distance $\frac{\ell}{2}$ from center of a square surface of side length ℓ. If the flux passing through the shaded region is then $\phi = \frac{5}{x\varepsilon_0}$, then write the value of x



Ans. **48.00**

Sol. flux passing through the complete square surface = $\frac{q}{6\varepsilon_0}$

8 triangular surface .... $\phi = \frac{5}{6\varepsilon_0}$

1 triangular surface .... $\frac{q}{48\varepsilon_0}$

5 triangular surface .... $\phi = \frac{q}{48\varepsilon_0} \times 5$

$$\phi = \frac{5q}{48\varepsilon_0} \text{ where } q = 1C = \frac{5}{48\varepsilon_0} = \frac{5}{x\varepsilon_0}$$

x = 48

14. Find minimum order of maxima of wavelength λ_1 on screen in YDSE where maxima of λ_1 = 480 nm coincide with maxima of λ_2 = 600 nm.

(1) 5 (2) 4 (3) 3 (4) 1

Ans. **(1)**

Sol. λ_1 = 480 nm

λ_2 = 600 nm

n_1 = ?

$y = n_1\lambda_1 = n_2\lambda_2$

$\Rightarrow \frac{n_1}{n_2} = \frac{600}{480} = \frac{5}{4}$

$n_1 = 5$

15. In a process pressure of the gas is directly proportional to temperature then choose correct option

(A) Process is isochoric

(B) Work done in process is zero

(C) Internal energy increases with increase in temperature

(1) A and B are correct (2) A and C are correct

(3) A, B and C are correct (4) B and C are correct

Ans. **(3)**

Sol. $P \propto T$ (volume is constant)

process is isochoric

work = $P\Delta V = 0$

$\Delta U = nC_v\Delta T$

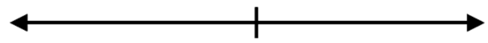
ΔU increase if temperature increase

16. A particle executes SHM with its time period 2 second and has amplitude of 1 cm. What is the ratio of total distance and displacement in 12.5 second :

(1) 25 : 1 (2) 5 : 1 (3) 4 : 5 (4) 3 : 2

Ans. **(1)**

Sol.



x = –1cm x = 0 x = 1 cm

12 second + 0.5 second

$$\text{Distance} = \frac{4}{2} \times 12 + \frac{4}{2} \times 0.5$$

$$= 24 + 1 = 25 \text{ cm}$$

Displacement = 1 cm

$$\frac{\text{Distance}}{\text{Displacement}} = \frac{25}{1}$$

17. Find the maximum possible velocity for the given angle of banking θ on a curved road of radius of curvature r having coefficient of friction μ.

(1) $v_{max} = \sqrt{\frac{gr(\mu + \tan\theta)}{(1 - \mu\tan\theta)}}$ (2) $v_{max} = \sqrt{\frac{gr(\mu - \tan\theta)}{(1 - \mu\tan\theta)}}$

(3) $v_{max} = \sqrt{\frac{gr(1 + \tan\theta)}{(1 - \mu\tan\theta)}}$ (4) $v_{max} = \sqrt{\frac{gr(\mu - \tan\theta)}{(1 + \mu\tan\theta)}}$

Ans. **(1)**

18. What is the fractional decrease in focal length of a lens when optical power is increased from 2.5 D to 2.6 D.

(1) 0.05 (2) 0.04 (3) 0.10 (4) 0.25

Ans. **(2)**

Sol. $f = \frac{1}{P}$

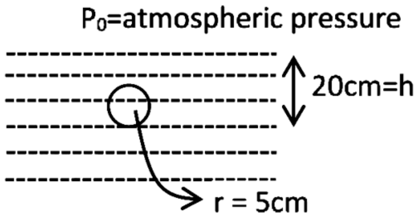
$$\frac{\Delta f}{f} = \frac{\Delta P}{P} = \frac{2.6 - 2.5}{2.5} = \frac{1}{25} = 0.04$$

19. A water bubble is at a depth of 20 cm and radius of bubble is 1 cm. If the inner pressure of the bubble is greater than the atmospheric pressure by 2100 N/m^2 then find the surface tension?

(1) 0.6 (2) 0.5 (3) 0.8 (4) 0.4

Ans. **(2)**

Sol.



$P_{in} - P_0 = 2100$ .... (i)

$P_{in} - P_0 = \frac{2T}{R} + h\rho g$ ...(ii)

From (i) and (ii)

$2100 = \rho gh + \frac{2T}{R}$

$\frac{2T}{R} = 2100 - \rho gh$

T = 0.5

20. If the distance between two parallel plates of a capacitor is d, A is the area of each plate, and E is the electric field between both the plates. Find the energy stored in capacitor.

(1) $\frac{1}{2}E^2A\epsilon_0 d$ (2) $\frac{1}{4}E^2A\epsilon_0 d$ (3) $\frac{3}{4}E^2A\epsilon_0 d$ (4) $E^2A\epsilon_0 d$

Ans. **(1)**

Sol. Energy density (u) = $\frac{1}{2}\epsilon_0 E^2$

Energy stored = energy density × volume

$= \frac{1}{2}\epsilon_0 E^2 \times (A \times d)$

$= \frac{1}{2}E^2A\,\epsilon_0\,d$

1. Difference of B.P and M.P in oxygen and sulpher can be explained by
(1) Atomicity (2) Atomic mass (3) Electronegativity (4) Electron gain enthalphy

Ans. **(1)**

Sol. The large difference between the melting and boiling points of oxygen and sulpher may be explained on the basis of their atomicity. O_2 exist as diatomic molcule where sulpher exist as polyatomic molecule S_8

2. Which of the following strong oxidising agent?
(1) Eu^{+2} (2) Ce^{2+} (3) Ce^{4+} (4) Eu^{4+}

Ans. **(4)**

Sol. M^{4+} will reduce itself to stable (+3) so, it will be good Oxidizing agent.

3. If 280 kg CO and 2320 kg of fe_3O_4 are made to react according to

$$fe_3O_4 + 4\,CO \rightarrow 4CO_2 + 3Fe$$

what is the weight of Fe produce (in kg)
Given : Mol. Wt. of CO and Fe_3O_4 are 28 and 232 u

Ans. **(420)**

Sol.

	fe_3O_4	+	4 CO	$\rightarrow$	3Fe + $4CO_2$
n_i	10000		10000 (LR)		
n_f					$10000 \times \frac{3}{4} = 7500$

$\therefore$ $W_{Fe} = 7500 \times 56$ g = 420 kg

4. A reaction is non spontaneous at freezing point and spontaneous at boiling point select the correct option
(1) Both ΔH and ΔS are positive (2) $\Delta H > 0$, $\Delta S < 0$
(3) $\Delta H < 0$, $\Delta S > 0$ (4) Both ΔH and ΔS are negative

Ans. **(1)**

Sol.

Case I	**Case II**
At freezing point	At boiling point
$\Delta G > 0$	$\Delta G < 0$
$\Delta G - T\Delta S > 0$	$\Delta H - T\Delta S < 0$
$\lvert\Delta G\rvert > \lvert T\Delta S\rvert$	$\lvert\Delta H\rvert < \lvert T\Delta S\rvert$

5. Standard potential
(i) $Fe^{3+} + 3e^- \rightarrow Fe$; E_1^0 = Z volt
(ii) $Ag^+ + 1e^- \rightarrow Ag$; E_2^0 = Y volt
(iii) $Fe^{2+} + 2e^- \rightarrow Fe$; E_3^0 = X volt
Find E° of reaction $Fe^{2+} + Ag^+ \rightarrow Fe^{3+} + Ag\,(s)$
(1) (Z – 2X + 3Y) volt (2) (X – 2Y + 3Z) volt (3) (Y – 2Z + 3X) volt (4) (Y – 3Z + 2X) volt

Ans. **(4)**

Sol. $Fe^{3+} + e^- \rightarrow Fe^{2+}$
(iv) = (i) – (iii)

$$\Delta G_4^0 = \Delta G_1^0 - \Delta G_3^0$$

$$-1.f.E_4^0 = -3.f.E_1^0 + 2.f.E_3^0$$

$$E_4^0 = 3E_1^0 - 2E_3^0 = 3Z - 2x$$

$$E^0 = E^0_{Ag^+/Ag} - E^0_{Fe^{2+}/Fe^{3+}} = Y - E^0_{Fe^{2+}/Fe^{3+}} = Y - 3Z + 2x$$

6. In a process pressure of gas is directly proportional to temperature then choose correct option
(A) Process is isochoric
(B) Work done in process is zero
(C) Internal energy increases with increase in temperature
(1) A and B are correct (2) A and C are correct
(3) A, B and C are correct (4) B and C are correct

Ans. **(3)**

Sol. $P \propto T$ (V, n = constant)

7. 1 mole of a complex with molecular formula CO $(NH_3)_5$ Cl_3 produces 3 mole ions upon complete ionisation. Upon adding excess $AgNO_3$, 2 mole AgCl are precipitated. Complex is:
(1) $[Co(NH_3)_5Cl]Cl_2$ (2) $[Co(NH_3)_4Cl_2]Cl.NH_3$
(3) $[Co(NH_3)_3Cl_3].2NH_3$ (4) $[Co(NH_3)_4Cl]Cl_2.NH_3$

Ans. **(1)**

Sol. $2Cl^-$ should be outside square bracket. Also, Co^{3+} shows CN = 6
$\therefore$ Complex : $[Co(NH_3)_5Cl]\ Cl_2$

8. Consider H_2O, NH_3, CH_4 and select correct statements:
(A) All are sp^3 hybridised
(B) Bond angle H–O–H, H–N–H, H–C–H are 104.5°, 107°, 109.5°
(C) Dipole moment: $CH_4 < NH_3 < H_2O$
(D) H_2O and NH_3 are lewis base and lewis acid
(E) NH_3 in H_2O is basic in nature
(1) A, B, C and E (2) A, B and D (3) A, C, D (4) B, D, E

Ans. **(1)**

Sol. Both NH_3 and H_2O behave as lewis base.

9. Which of the following is not true combination from given statement.
(a) Elements in periodic table are linearly arranged with atomic weight
(b) Elements in periodic table are linearly arranged with atomic number
(c) Element having similar electronic configuration are arranged in same group
(d) Using periodic table we can identify in which subshell last electron enters
(e) Isotopes of an element are placed in periodic table.
(1) a, e only (2) b, e only (3) b, c, d only (4) a, c, d, e only

Ans. **(1)**

10. If the k_{sp} of $Cr(OH)_3$ is $1.6 \times 10^{-30}\ M^4$ the molar solubility of salt in water is
(1) $\left(\frac{1.6\times10^{-30}}{27}\right)^{1/4}$ (2) $\left(\frac{16}{27}\times10^{-30}\right)^{1/4}$ (3) $\left(\frac{160}{27}\times10^{-30}\right)^{1/4}$ (4) None of these

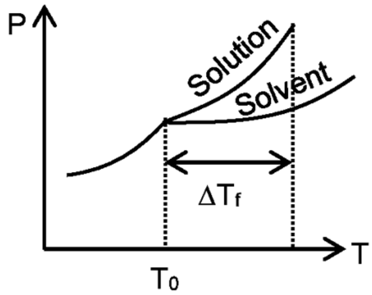
Ans. **(1)**

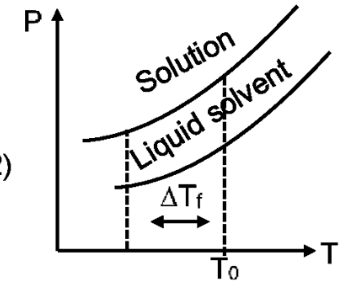
Sol. $Cr(OH)_3\,(s) \rightleftharpoons \underset{s}{Cr^{3+}} + \underset{3s}{3OH^-}$

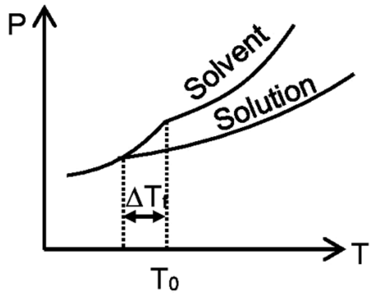
$k_{sp} = [Cr^{3+}]\,[OH^-]^3 = s(3s)^3$
$= 27\,S^4$

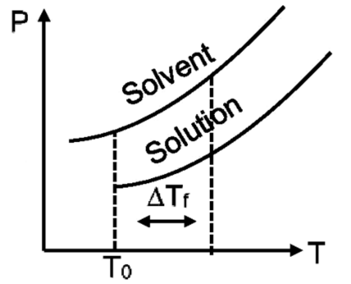
$S = \left(\frac{1.6\times10^{-30}}{27}\right)^{1/4}$

11. Select the correct graph.

(1) 

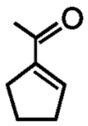
(2) 

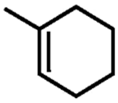
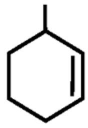
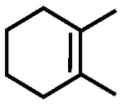
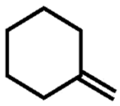
(3) 

(4) 

Ans. **(3)**

Sol. T at which $VP_{solid} = VP_{liquid}$ is freezing point of solution is less than that of solvent.

12.  is formed by ozonolysis followed by aldol condensation of Alkene. Alkene can be:

(1)  (2)  (3)  (4) 

Ans. **(1)**

13. Ribose present in DNA

(A) It is a pentose sugar

(B) It is a present in pyranose form

(C) It is present in D-configuration

(D) It is reducing sugar in free form

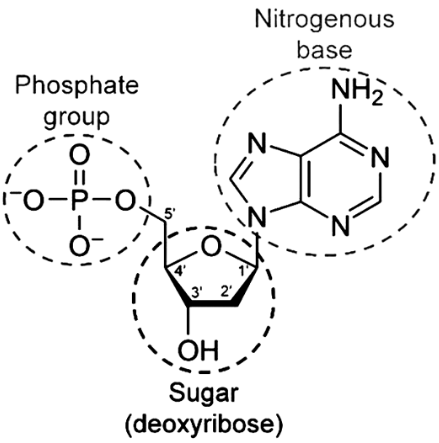
(E) α anomeric form is present.

Correct options are :

(1) A, C, D (2) A, B, D (3) A, B, C, D, E (4) A, C, E

Ans. **(1)**

Sol.



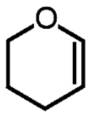
14. Arrange following for reaction rate with nucleophilic attack

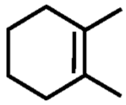
(a) Acetophenone
(b) p-tolylaldehyde
(c) Benzaldehyde
(d) p-Nitrobenzaldehyde

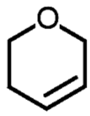
(1) d > c > b > a (2) a > c > b > d (3) d > b > c > a (4) d > a > b > c

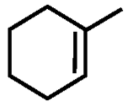
Ans. (1)

15. Which compound react fastest with HBr?

(1) 

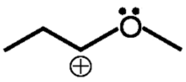
(2) 

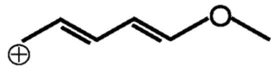
(3) 

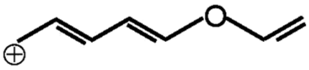
(4) 

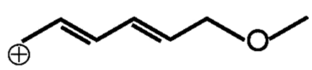
Ans. (1)

16. Stability of carbocation is maximum is?

(1) 

(2) 

(3) 

(4) 

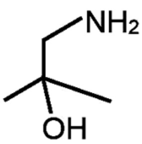
Ans. (2)

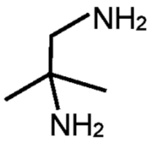
17. **Statement-I :** Dumas method is used for estimation of Nitrogen.
Statement-II : In Dumas method Nitrogen present in compound is converted to $(NH_4)_2SO_4$

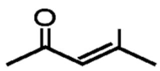
(1) Both Statement I and statement II are true
(2) Both statement I and statement II are false
(3) Statement I is true but statement II is false
(4) Statement I is false but statement II is true

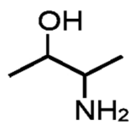
Ans. (3)

18. $CH_3–C≡CH \xrightarrow[\substack{2.HCN/HO^{\ominus} \\ 3.H_2/Ni}]{1.HgSO_4}$

(1) 

(2) 

(3) 

(4) 

Ans. (1)

19. **Statement-I :** $CH_3–CH_2–CH_2–CH_2–Cl + OH^{\ominus} \longrightarrow$ Reaction is favoured in less polar solvent.
Statement-II : $CH_3–CH_2–CH_2–CH_2–Cl + R_3N \longrightarrow$ Reaction is favoured in more polar solvent.

(1) Both Statement I and statement II are true
(2) Both statement I and statement II are false
(3) Statement I is true but statement II is false
(4) Statement I is false but statement II is true

Ans. (1)

Question:If the 5^{th}, 6^{th}, and 7^{th} term of the binomial expansion of $(1 + x^2)^{n+4}$ are in A.P. Then the greatest binomial coefficient in the expansion of $(1 + x^2)^{n+4}$ is

Options:

(a) 10

(b) 35

(c) 25

(d) 14

Answer: (b)

$^NC_4{}^NC_5,{}^N C_6 \rightarrow AP, \quad N = n + 4$

$^NC_4 +^N C_6 = 2.{}^NC_5$

$\Rightarrow \frac{^NC_4}{^NC_5} + \frac{^NC_5}{^NC_5} = 2$

$\Rightarrow \frac{5}{N-4} + \frac{N-5}{6} = 2$

$\Rightarrow 30 + n^2 - 9N + 20 = 12N - 98$

$\Rightarrow N^2 - 21N + 98 = 0$

$\Rightarrow (N - 7)(N - 14) = 0 \Rightarrow N = 7, 14$

Greatest Binomial Coefficient $=^7 C_3 =^7 C_4 = \frac{7\times6\times5}{6} = 35$

or $^{14}C_7$

Question: The number of 3 digit numbers which is divisible by 2 and 3 but not divisible by 4 and 9.

Options:

(a) 150

(b) 25

(c) 125

(d) 50

Answer: (d)

Divisibe by 2 but not by 4 = 225

102, 106, 110,......998

out of this divisible by 3

102, 114, 126, ...990

$12n + 90 = n = 1, 2, ...75$

So only divisible by 3 but not by 9

n = 1, 2, 4, 5,7, 8,......i.e., 50

Question: If A is 3 × 3 matrix such that det(A) =2. Then det

$(adj(adj(adj(adjA))))$

Options:
(a) 2^{32}
(b) 2^{16}
(c) 2^{8}
(d) 2^{12}
Answer: (b)

$$|A| = 2$$

$$||adj(adj(ajd(adjA)))||$$

$$= |A|^{24} = 2^{16}$$

Question:Evaluate $\lim_{x\to 0} \cos ecx.\left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}\right)$
Options:
(a) 1
(b) 0
(c) $\frac{1}{2\sqrt{5}}$
(d) $-\frac{1}{2\sqrt{5}}$
Answer: (d)

$$\lim_{x\to 0} \frac{\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}}{\sin x}$$

$$\lim_{x\to 0} \frac{\frac{1}{2\sqrt{2\cos^2 x 3\cos x}}[(4\cos x)(-\sin x) - 3\sin x] - \frac{1}{2\sqrt{\cos^2 x + \sin + 4}}[(2\cos x)(\sin x) + \cos x]}{\cos x}$$

$$= 0 - \frac{1}{2\sqrt{5}} = -\frac{1}{2\sqrt{5}}$$

Question:If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c}$ is **coplanar with** $\vec{a}$ and $\vec{b}$. Also $\vec{a}.\vec{c} = 5$ and $\vec{c}$ is pependicular to $\vec{b}$. Then $|\vec{c}|$ is
Options:
(a) 18
(b) 16
(c) $\frac{\sqrt{5}}{14}$
(d) $\sqrt{\frac{11}{6}}$
Answer: (d)

$\vec{a} = (1, 2, 3), \vec{b} = (3, 1, -1), a.\ c = 5$

$\vec{c} = \lambda\vec{b} \times \left(\vec{a} \times \vec{b}\right)$

$= \lambda\left[b^2\vec{a} - \left(\vec{b}\cdot\vec{a}\right)\vec{b}\right]$

$= \lambda\left(11\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - (2)\left(3\hat{i} + \hat{j} - \hat{k}\right)\right)$

$= \lambda\left(5\hat{i} + 20\hat{j} + 35\hat{k}\right)$

$= 5\lambda\left(\hat{i} + 4\hat{j} + 7\hat{k}\right)$

$\vec{a}\cdot\vec{c} = 5 \Rightarrow 5\lambda(1 + 8 + 21) = 5$

$\Rightarrow 5\lambda = \frac{1}{6}$

$\left|\vec{c}\right| = 5\lambda\sqrt{66} = \frac{\sqrt{66}}{6} = \sqrt{\frac{66}{36}} = \sqrt{\frac{11}{6}}$

Question:The area of the region bounded by S(x, y) such that S= {(x, y) : $x^2 + 4x + 2 \le y \le |x + 2|$} is (in sq. units)

Options:

(a) $\frac{24}{5}$

(b) 5

(c) $\frac{20}{3}$

(d) 7

Answer: (c)

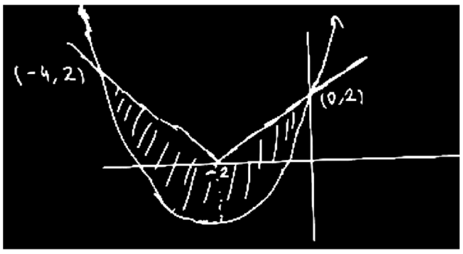
$(x+2)^2 - 2 \le y \le |x+2|$

$A = \int_{-4}^{-2}\left(-x - 2 - x^2 - 4x - 2\right)dx + \int_{-2}^{0}\left(x + 2 - x^2 - 4x - 2\right)dx$

$= \int_{-4}^{-2}\left(-x^2 - 5x - 4\right)dx + \int_{-2}^{0}\left(-x^2 - 3x\right)dx$

$= \left(-\frac{x^3}{3} - \frac{5x^2}{2} - 4x\right)_{-4}^{-2} + \left(\frac{-x^3}{3} - \frac{3x^2}{2}\right)_{-2}^{0}$

$= \frac{10}{3} + \frac{10}{3} = \frac{20}{3}$



Question: If $\dfrac{dy}{dx} + \left(\dfrac{x}{1+x^2}\right)y = \dfrac{\sqrt{x}}{\sqrt{1+x^2}}; y(0) = 0,$ **then y(1) will be**

Options:

(a) $\frac{2}{3}$

(b) $\frac{2}{\sqrt{3}}$

(c) $\frac{\sqrt{2}}{3}$

(d) $\sqrt{\frac{2}{3}}$

Answer: (c)

$\frac{dy}{dx} - \frac{x}{1+x^2}y = \frac{\sqrt{x}}{\sqrt{1+x^2}}, P = \frac{-x}{1+x^2}, Q = \sqrt{\frac{x}{1+x^2}}$

$I.F = e^{\int -\frac{x}{1+x^2}}$

Let $1 + x^2 = t, 2xdx = dt, -xdx = -\frac{dt}{2}$

So I.F $= e^{-\frac{1}{2}\int \frac{1}{2}dt} = e^{-\frac{1}{2}\log t} = \sqrt{t} = \sqrt{1+x^2}$

Now y.I.F $= \int \sqrt{\dfrac{x}{1+x^2}} \times \sqrt{1+x^2}dx$

$y.\sqrt{a+x^2} = \int \sqrt{x}dx = \dfrac{2}{3}x^{\frac{3}{2}} + c$

$y(0) = 0$ so $O = C$

$y(1) = y.\sqrt{2} = \frac{2}{3} \times 1 + 0$

$y = \frac{\sqrt{2}}{3}$

Question:If α and β are real numbers such that $\sec^2(\tan^{-1}(\alpha) + \operatorname{cosec}^2(\cot^{-1}(\beta)) = 36$ and $\alpha + \beta = 8$, then $(\alpha^2 + \beta)$ is $(\alpha > \beta)$

Options:

(a) 23
(b) 28
(c) 24
(d) 27

Answer: (b)

$\sec^2\left(\tan^{-1}\alpha\right) + \cos ec^2\left(\cot^{-1}\beta\right) = 36, \quad \alpha + \beta = 8$

$1 + \alpha^2 + 1 + \beta^2 = 36 \Rightarrow \alpha^2 + \beta^2 = 34$

$\Rightarrow \alpha^2 + (8-\alpha)^2 = 34$

$\Rightarrow 2\alpha^2 - 16\alpha + 30 = 0$

$\alpha^2 - 8\alpha + 15 = 0 \Rightarrow \alpha = 5, \beta = 3$

$\alpha^2 + \beta = 28$

Question: $f(x) - 6f\left(\dfrac{1}{x}\right) = \dfrac{35}{3x} - \dfrac{5}{2}$. **If** $\lim\limits_{x\to 0}\left(\dfrac{1}{\alpha x} + f(x)\right) = \beta$. **find $(\alpha + 2\beta)$.**

Solution:

$f(x) - 6f(\frac{1}{x}) = \frac{35}{3x} - \frac{5}{2}$

$6f(\frac{1}{x}) - 36f(x) = (\frac{35x}{3} - \frac{5}{2}) \times 6$

$-35f(x) = \frac{35}{3x} - \frac{5}{2} + 70x - 15$

$-35f(x) = 70x + \frac{35}{3x} - \frac{35}{2}$

$f(x) = \frac{1}{2} - 2x - \frac{1}{3x}$

$\underset{x\to 0}{\text{lt}} \frac{1}{\alpha x} + \frac{1}{2} - 2x - \frac{1}{3x}$

$= (\frac{1}{\alpha} - \frac{1}{3}) + \frac{1}{2} - 2x$

$\alpha = 3$

$\beta = \frac{1}{2}$

Question: $I_{mn} = \displaystyle\int_0^1 x^{m-1}(1-x)^{n-1}dx$**, then I(9, 13) is equal to**

Solution :

$I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}dx$

$I_{9,13} = \int_0^1 x^8(-x)^{12}dx$

$= x^8 \frac{(1-x)}{-13}\Big|_0^1 - \int_0^1 8x^7 \frac{(1-x)^{13}}{-13}dx$

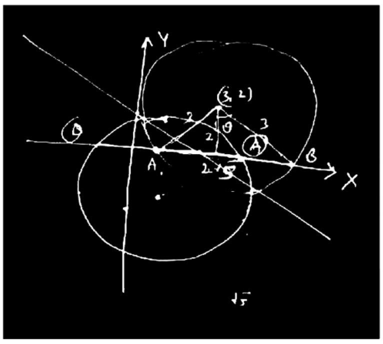
$= \frac{8}{13}\int_0^1 x^7(1-x)^{13}dx$

$= \frac{8!}{13-14....20}\int_0^1 (1-x)^{20}dx$

$= \frac{1}{{}^{20}C_8} \times \frac{1}{21}$

Question:Consider the circle $x^2 + y^2 - 2x + 4y - 4 = 0$. This circle is reflected about the line $x + 2y = 2$. A chord of this reflected circle through origin and parallel to x-axis meets the circle at A and B. Find the area of region bounded by AB and circle (smaller one).

Solution :



$(x-3)^2 + (y-2)^2 = 9$

$(x-3)^2 = 5$

$x = 3 \pm \sqrt{5}$

$m = \sin\theta = \frac{\sqrt{5}}{3}$

$A = x\theta.\,3^2 - \frac{1}{2} \times 2\sqrt{5} \times 2$

$= \left[9\sin^{-1}\frac{\sqrt{5}}{3} - 2\sqrt{5}\right]$

Question:A and C are two points on the line $\dfrac{x-6}{3} = \dfrac{y-7}{2} = \dfrac{z-7}{-2}$ **such that AC = 6. B is (1, 1, -2). Find area of ΔABC**

Solution :

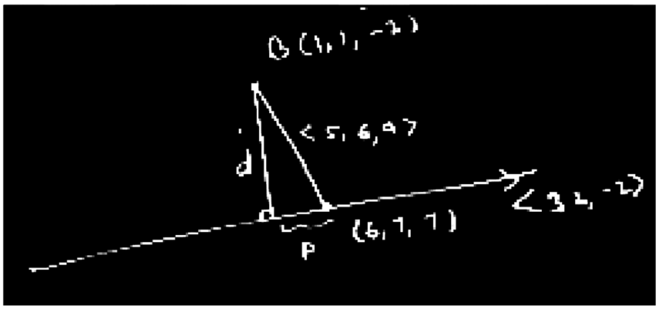
$P = \frac{15+12-18}{\sqrt{17}}$

$= \frac{9}{\sqrt{17}}$

$d = \sqrt{142 - \frac{81}{17}} = \sqrt{\frac{2333}{17}}$

$A = \frac{1}{2} \times 6 \times \sqrt{\frac{2333}{17}}$

35.13



Question: Let the parabola $y = x^2 + px - 3$ cuts the coordinate axes at P, Q and R. A circle with centre (-1, -1) passes through P, Q and R, then the area of triangle PQR.

Solution :

$R(0,-3) r = \sqrt{5}$ $\quad$ $\mathrm{P}(\alpha,0) Q(\beta,0)$

$(x+1)^2 + (y+1)^2 = 5$

$(\alpha+1)^2 + 1 = 5$

$(\alpha+1)^2 = 4$

$\alpha = 1, -3$

$P(1,0), Q(-3,0)$

$\text{Area} = \frac{1}{2} \times 4 \times 3 = 6$

Question:Find the product of all real roots of equation $(x^2 - 9x + 11)^2 - (x - 4)(x - 5) = 2$ is

Solution :

$\left(x^2-9x+11\right)^2-\left(x^2-9x+20\right)=2$

$(t+11)^2-(t+20)=2, t=x^2-9x\geq\frac{-81}{4}$

$t^2\_21t+99-0\Rightarrow t=\frac{-21\pm3\sqrt{5}}{2}=-7.14,-13.8$

Product of roots $=99$

Question: For a distribution of 10 observations, $\sum_{i=1}^{10}x_i=55$ and $\sum_{i=1}^{10}x_i^2=328$. If the observations 4 and 5 are replaced by 6 and 8 respectively, then the new variance is

Options:

(a) 2.5

(b) 2.7

(c) 3.4

(d) 3.6

Answer: (b)

$$\sum x=55-4-5+6+8=60$$

$$\sum x^2=328-16-25+36+64=387$$

$$\overline{x}=6$$

$$\sigma^2=\frac{387}{10}-6^2=38.7-36=2.7$$

Question: A and B playing a game (throwing a pair of dice alternatively). A wins the game when sum = 5 and B wins the game when sum = 8. Probability of A winning given that A starts the game.

Solution :

$$P(A)=\frac{4}{36}=\frac{1}{9},P(B)=\frac{5}{36}$$

$$P(Awins)=\frac{4}{36}+\frac{32}{36}\cdot\frac{31}{36}\cdot\frac{4}{36}+......to\propto$$

$$=\frac{\frac{4}{36}}{2-\frac{32}{36}\cdot\frac{31}{36}}=\frac{\frac{1}{9}}{1-\frac{8}{9}\cdot\frac{31}{36}}=\frac{\frac{1}{9}}{1-\frac{62}{81}}$$

$$=\frac{9}{19}$$

Question:If the images of the points A(1,3), B(3,1) and C(2,4) in the line x + 2y = 4 are D, E and F respectively, then the centroid of the triangle DEF is

Solution :

The mirror line is $x+2y-4=0$

image of A $(1{,}3)$ is $\frac{x-1}{1}=\frac{y-3}{2}=-2\left(\frac{1+6-4}{5}\right)$

image of B $(3,1)$ is $\frac{x-3}{1}=\frac{y-1}{2}=-2\left(\frac{3.2.4}{5}\right)$

image of $(2,4)$ is $\frac{x-2}{1}=\frac{y-4}{2}=-2\left(\frac{2+8-4}{5}\right)$

$x=\frac{-2}{5}, y=-\frac{4}{5}$

So $D=\left(-\frac{1}{5},\frac{3}{5}\right), E=\left(\frac{13}{5},\frac{1}{5}\right), F=\left(\frac{-2}{5},\frac{-4}{5}\right)$

Centroid $=\left(\frac{\frac{-1}{5}+\frac{13}{5}\frac{-2}{5}}{3},\frac{\frac{3}{5}+\frac{1}{5}-\frac{4}{5}}{3}\right)$

$=\left(\frac{10}{15},\frac{10}{15}\right)=\left(\frac{2}{3},0\right)$