Memory Based Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2025 (Online) Phase-2

(Physics, Chemistry and Mathematics)

29 Jan 2025 (Evening Shift)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains **Three Parts**. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is **Mathematics**. Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

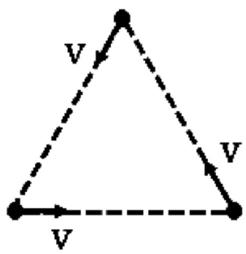
(5) **Section - B :** Attempt all questions.

(6) **Section - A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section - B (21 – 25) contains** 5 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

SECTION-A

1. Three identical particles, each of mass m move under the influence of mutual attraction forces. Initially they are on the vertices of an equilateral triangle of side 'a' and have equal speed v directed towards the adjacent particles as shown. The net angular momentum about the centre just before collision is :



(1) $\frac{3\sqrt{3}}{2}(mva)$ (2) $\frac{3}{2}(mva)$

(3) $\frac{\sqrt{3}}{2}(mva)$ (4) $\frac{1}{\sqrt{3}}(mva)$

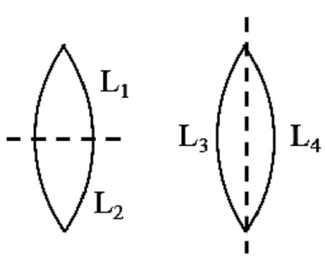
Ans. **(3)**

Sol. $d = \frac{a}{2\sqrt{3}}$

$$\left|\vec{L}\right| \times 3 = mv\frac{a}{2\sqrt{3}}(3)$$

$$= \frac{mva\sqrt{3}}{2}$$

2. An equiconvex lens is cut in two ways as shown. If the focal length of the parts are as mentioned in the diagram. Find $\frac{L_1}{L_3}$.



(1) 2 (2) 4

(3) $\frac{1}{2}$ (4) $\frac{1}{4}$

Ans. **(3)**

Sol. $\frac{1}{f} = (\mu - 1)\frac{2}{R}$ $\frac{f_3}{f_1} = 2$ $\frac{f}{f_3} = \frac{1}{2}$

$$\frac{1}{f_3} = (\mu - 1)\frac{1}{R}$$

3. Match the physical quantities with their corresponding dimensions

Column-I		Column-II	
(A)	Young's modulus	(i)	$[AL^2]$
(B)	Magnetic moment	(ii)	$[ML^2T^{-2}A^{-1}]$
(C)	Magnetic flux	(iii)	$[MT^{-2}A^{-1}]$
(D)	Magnetic intensity	(iv)	$[ML^{-1}T^{-2}]$

(1) A–(iii), B–(i), C– (ii), D–(iii)
(2) A–(iv), B–(i), C– (ii), D–(iii)
(3) A–(iii), B–(i), C– (ii), D–(iv)
(4) A–(iii), B–(ii), C– (i), D–(iv)

Ans. **(2)**

Sol. (A) $[Y] = [ML^{-1}T^{-2}]$

(B) $\left|\vec{M}\right| = i_{Area} = \left[AL^2\right]$

(C) $\phi = BA = \left[MT^{-2}A^{-1}L^2\right]$

(D) $[B] = \left[\frac{F}{2v}\right] = \frac{MLT^{-2}}{ATLT^{-1}}$

$$= \left[MT^{-2}A^{-1}\right]$$

4. Two particles A and B of same mass are performing SHM vertically with two different springs of spring constants K_1 and K_2. If amplitude of both is same. Find ratio of the maximum speed of A to B.

(1) $\sqrt{\frac{K_2}{K_1}}$ (2) $\sqrt{K_2K_1}$

(3) $\sqrt{\frac{K_1}{K_2}}$ (4) $\sqrt{\frac{K_1+K_2}{K_1-K_2}}$

Ans. **(3)**

Sol. $\frac{(v_A)_{max}}{(v_B)_{max}} = \frac{A_1\omega_1}{A_2\omega_2} = \frac{\omega_1}{\omega_2}$

$= \sqrt{\frac{k_1}{k_2}} \ \{A_1 = A_2\}$

5. A physical quantity Q is given as $Q = \frac{ab^4}{cd}$, if the percentage error is a, b, c and d are 2%, 1%, 2% and 1% respectively, then the percentage error in Q will be :

(1) 10% (2) 9%

(3) 2% (4) 1%

Ans. **(2)**

Sol. $\frac{\Delta Q}{Q} = \frac{\Delta a}{a} + \frac{4\Delta b}{b} + \frac{\Delta c}{c} + \frac{\Delta d}{d}$

$= 9\%$

6. A circular plate capacitor has constant current 0.15 Amp flowing through it. Potential across capacitor varies as 7×10^8 V/s. Find distance between two circular plates of capacitor. (Radius of circular plate = 10cm, $\pi = \frac{22}{7}$)

(1) 1.29 cm (2) 1.29 mm

(3) 2.58 mm (4) 2.58 cm

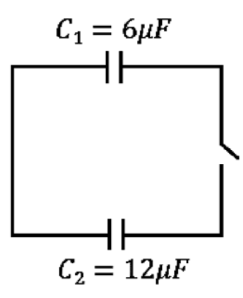
Ans. **(2)**

Sol. $i = C\frac{dv}{dt}$

$0.15 = \frac{\pi r^2 \varepsilon_0}{d} 7 \times 10^8$

$d = 1.29mm$

7. A capacitor $C_1 = 6\mu F$, initially charged with a cell of emf 5V is disconnected and connected to another capacitor $C_2 = 12\mu F$ which is initially uncharged. Find final charges on C_1 and C_2



(1) 0μC, 30μC (2) 10μC, 20μC

(3) 20μC, 10μC (4) 30μC, 0μC

Ans. **(2)**

Sol. 30 = v [6+12]

$v = \frac{30}{18}$

$Q_1 = 6 \times \frac{30}{18} = 10\mu C$

$Q_2 = 12 \times \frac{30}{18} = 20\mu C$

8. Statement 1: On increasing the pressure, the volume decrease is more in an isothermal process than in an adiabatic process.

Statement 2: Adiabatic process is given by PV^γ = constant.

(1) Statement-1 is true and Statement-2 is false

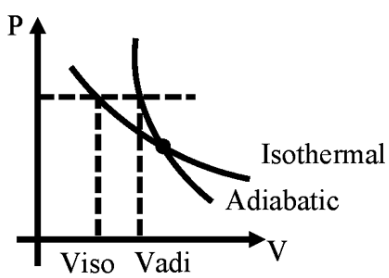
(2) Statement-1 is false and Statement-2 is true

(3) Both Statements are true

(4) Both Statements are false

Ans. **(3)**

Sol.



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9. Two planet A and B are revolving around a massive star such that $r_A = 2r_B$ and $m_A = 4\sqrt{3}m_B$. Find ratio of angular momentum of planet B to planet A.

(1) $8\sqrt{3}$ (2) $\frac{1}{4\sqrt{6}}$

(3) $\frac{1}{4\sqrt{3}}$ (4) $\frac{2}{\sqrt{6}}$

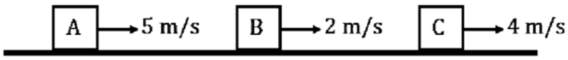
Ans. **(2)**

Sol. $L_A = M_A\sqrt{\left(\frac{GM}{r_A}\right)}r_A$

$L_B = \frac{M_B}{4\sqrt{3}}\sqrt{\left(\frac{GM_2}{r_A}\right)}\frac{r_A}{2}$

$\frac{L_B}{L_A} = \frac{1}{4\sqrt{6}}$

10. Three blocks of same mass set in motion as shown. (all collisions are elastic)



Statement-1 : After all collisions velocities of A, B, C are 4 m/s, 2 m/s and 5 m/s respectively.

Statement-2 : Velocities get interchanged in elastic collision of blocks of same mass.

(1) Statement-1 is true and Statement-2 is false

(2) Statement-1 is false and Statement-2 is true

(3) Both Statements are true

(4) Both Statements are false

Ans. (2)

Sol.

5m/sec 2m/sec 4m/sec

2m/sec 5m/sec 4m/sec

2m/sec 4m/sec 5m/sec

11. An electromagnetic wave propagates in +X-direction. Then, electric field and magnetic field are directed along

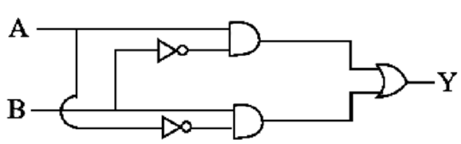
(1) X, Y (2) Y, Z

(3) Z, Y (4) Y, X

Ans. (2)

Sol. Theoretical

12. The truth table for the logical circuit shown below is



(1)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(2)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(4)

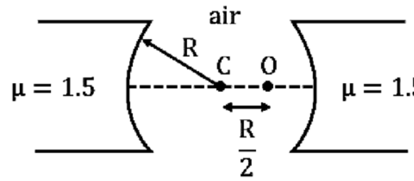
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Ans. (2)

Sol. $\left(A\overline{B}\right)+\left(\overline{A}B\right)=Y$

XOR gate

13. Figure shows two spherical surfaces of radius R having common centre. If the object is placed at O, find the distance between the first images formed by both the surfaces.



(1) $\frac{4R}{27}$ (2) $\frac{4R}{35}$

(3) $\frac{8R}{35}$ (4) $\frac{4R}{70}$

Ans. (2)

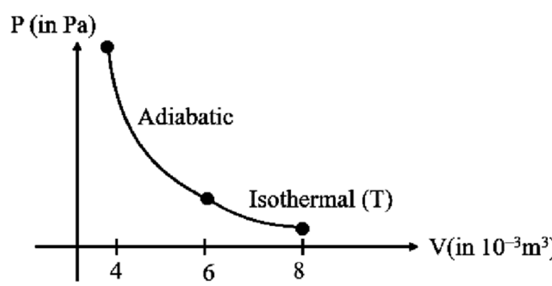
Sol. $\frac{3}{2v_1}-\frac{1}{-\frac{3}{2}R}=\frac{\frac{3}{2}-1}{-R}$ $\quad$ $\frac{3}{2v_2}-\frac{1}{-\frac{R}{2}}=\frac{\frac{3}{2}-1}{-R}$

$v_1=\frac{-9}{7}R$ $\quad$ $v_2=\frac{-3R}{5}$

$d=2R-|v_1|-|v_2|$

$d=\frac{4}{35}R$

14. P-V graph is given as shown in figure for one mole of an ideal gas



Find the total heat supplied to the system :

(1) $RT\ell n\left(\frac{4}{3}\right)$ (2) $RT\ell n\left(\frac{9}{2}\right)$

(3) $RT\ell n\left(\frac{8}{9}\right)$ (4) $RT\ell n\left(\frac{16}{3}\right)$

Ans. (1)

Sol. $\Delta Q_{adiabatic}=0$

$\Delta Q=\Delta W$ [in isothermal]

$= nRT\ \ln\left(\frac{v_f}{v_i}\right)$

$\Delta Q=(1)RT\ln\left(\frac{8}{6}\right)$

15. If sand falls on a conveyer belt moving with speed v such that $\frac{dm}{dt} \propto \sqrt{v}$. Find how the power required to keep the speed of belt constant as v varies with v.

(1) v^2 (2) $v^{5/2}$

(3) v (4) Independent of v

Ans. **(2)**

Sol. $F_m = v_{rel}\frac{dm}{dt}$

$= kv^{3/2}$

$P = F_m v = kv^{5/2}$

16. A cup of coffee take a time 't' to cool from 90°C to 80°C in a surrounding of 20°C. If a similar cup coffee is cooled from 80°C to 60°C in the same surrounding, it takes time

(1) $\frac{13t}{5}$ (2) $\frac{5t}{13}$

(3) $\frac{12t}{5}$ (4) 2t

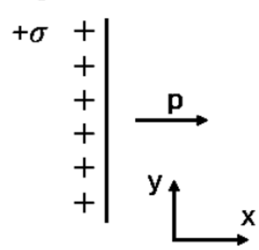
Ans. **(1)**

Sol. $\frac{10}{t} = k(85-20)$

$\frac{20}{t'} = k(70-20)$

$t' = \frac{13t}{5}$

17. A dipole is placed such that its axis is perpendicular to an infinite charged sheet. Select the correct options :



(A) $\tau_{net} = 0$, F_{net} is along –ve x-axis

(B) $\tau_{net} = 0$, U = min

(C) $\tau_{net} = 0$, $F_{net} = 0$

(D) τ_{net} and U both are maximum

(1) (A), (B), (C) and (D)

(2) (B) and (C)

(3) (A) and (C)

(4) (B) and (D)

Ans. **(2)**

Sol. Theoretical

18. Choose the correct option :

(1) Stopping potential $= \frac{1}{e}(K_{max})$

(2) Stopping potential increases if intensity of incident beam increases

(3) Stopping potential increases if intensity of incident beam decreases

(4) Stopping potential increases if wavelength of incident beam increases

Ans. **(1)**

Sol. Theoretical

SECTION-B

1. A solenoid of radius 10 cm carrying current 0.29 A and having total 200 turns. If magnetic field inside solenoid is 2.9×10^{-4} T. Length of solenoid is nπ cm, report n.

Ans. **(8)**

Sol. $\mu_0 ni = B$

$n = N/L$

$L = 8\pi cm$

2. A converging lens of focal length 24 cm, made of glass ($\mu_{glass} = \frac{3}{2}$) is immersed completely in water ($\mu_{water} = \frac{4}{3}$). It will now behave like a converging lens of focal length _______ cm.

Ans. **(96)**

Sol. $\frac{1}{24} = (1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f} = \left(\frac{1.5}{1.33} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$f = 96\ cm$

3. Find the number of spectral lines formed in a sample of Hydrogen atom in which highest orbit of electrons is 4.

Ans. **(6)**

Sol. ${}^4C_2 = 6$

SECTION-A

1. Which element in group-15 has lowest ionisation energy.

(1) Bi (2) P

(3) As (4) Sb

Ans. **(1)**

Sol. I.E. order: N > P > As > Sb > Bi

(in KJ/mole) 1402 1012 947 834 703

2. In which of the following O_2 will release at anode on electrolysis.

(A) $AgNO_{3(aq)}$ using silver electrodes

(B) $AgNO_{3(aq)}$ using pt electrodes

(C) Very dilute H_2SO_4

(D) Very concentrated H_2SO_4

(1) A, B (2) B, C

(3) B, D (4) A,D

Ans. **(2)**

Sol. **$AgNO_3$(aq)** (using inert electrodes)

Cathode : $Ag^+ + e^- \rightarrow Ag(s)$

Anode : $2H_2O \rightarrow O_2 + 4H^+ + 4e^-$

Very dilute H_2SO_4

Cathode : $2H^+ + 2e^- \rightarrow H_2(g)$

Anode : $2H_2O \rightarrow O_2 + 4H^+ + 4e^-$

3. Consider the following thermochemical reactions and choose the correct option.

C (diamond) $\rightarrow$ C (graphite) + x kJ

C (diamond) + $O_2 \rightarrow CO_2$ + y kJ

C (graphite) + $O_2 \rightarrow CO_2$ + z kJ

(1) $x = y + z$ (2) $x = y - z$

(3) $x + y = z$ (4) $x + y = -z$

Ans. **(2)**

Sol. (I) $C_{diamond} \rightarrow C_{graphite}$: $\Delta H_1 = -x$ kJ

(II) $C_{diamond} + O_2 \rightarrow CO_2(g)$ + y kJ : $\Delta H_2 = -y$ kJ

(III) $C_{graphite} + O_2 \rightarrow CO_2(g)$ + z kJ : $\Delta H_3 = -z$ kJ

Eqn (1) is obtained by (II) – (III)

$\therefore \Delta H_1 = \Delta H_2 - \Delta H_3$

$\Rightarrow -x = -y - (-z)$

$\Rightarrow x = y - z$

4. A drug becomes ineffective when it decomposes to 50 % of its concentration. If 16 mg of said drug becomes 4 mg in 12 months, find the time in which drug becomes ineffective given that decomposition of drug follows first order kinetics.

(1) 6 months (2) 3 months

(3) 2 months (4) 12 months

Ans. **(1)**

Sol.

$$16 \text{ gram} \xrightarrow{t_{1/2} = 6 \text{ months}} 8 \text{ gram} \xrightarrow{t_{1/2} = 6 \text{ months}} 4 \text{ gram}$$

12 months

5. **Statement-1:** On mixing NaCl in ice, it will not melt at 0°C.

Statement-2: On mixing NaCl freezing point of ice decreases.

(1) Statement 1 and statement 2, both are correct.

(2) Statement 1 is correct and statement 2 is incorrect.

(3) Statement 1 is incorrect and statement 2 is correct.

(4) Statement 1 and statement 2, both are incorrect.

Ans. **(3)**

Sol. On mixing NaCl in ice, its freezing point decreases and it will melt below 0° C

6. Predict value of magnetic moment of given co-ordination compound $K_3[Fe(OH)_6]$ and $K_4[Fe(OH)_6]$

(1) 5.91 BM, 4.9 BM

(2) 4.9 BM, 5.91 BM

(3) 4.9 BM, 3.87 BM

(4) 3.87 BM, 4.9 BM

Ans. **(1)**

Sol. **$K_3[Fe(OH)_6]$**

$Fe^{+3} \rightarrow [Ar]3d^5$

$t_{2g}^3 e_g^2$

No. of unpaired $e^- = 5$

$\mu = \sqrt{5 \times 7} = \sqrt{35}$BM

$\mu = 5.91$ BM

$K_4[Fe(OH)_6]$

$Fe^{+2} \rightarrow [Ar]3d^6$

$t_{2g}^4 e_g^2$

No. of unpaired $e^- = 4$

$\mu = \sqrt{4 \times 6} = \sqrt{24}$ BM

$\mu = 4.9$ BM

7. In which of the following, oxides of given metals will have minimum radius?

Li, Be, B, Na, Al, Mg

(1) A_2O (2) AO_2

(3) A_2O_3 (4) AO

Ans. **(4)**

Sol. 'Be' metal will have minimum radius,

So oxide will be BeO

8. **Assertion:** On increasing the pressure, the volume decrease is more in an isothermal reversible process than in an adiabatic reversible process.

Reason : Adiabatic reversible process is given by PV^γ, and isothermal reversible process is given by PV = constant.

(1) Assertion is correct and reason is incorrect.

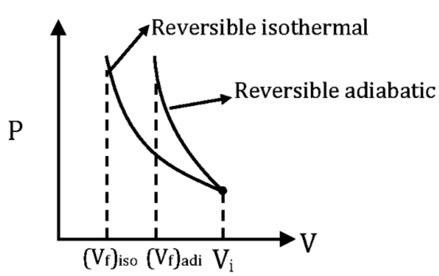
(2) Assertion and reason both are correct.

(3) Assertion is incorrect and reason is correct.

(4) Assertion and reason, both are incorrect.

Ans. **(2)**

Sol.



In Adiabatic reversible process temperature increases while in isothermal reversible process temperature is constant.

Hence decrease in volume is more in reversible isothermal process.

9. Correct statements for 0.1 M solution of KI

$5I^- + IO_3^- + 6H^+ \longrightarrow 3I_2 + 3H_2O$

I. 200 ml KI reacts with 0.004 mole of KIO_3

II. 200 ml KI reacts with 0.006 mole of H_2SO_4

III. 0.1 litre KI will give 0.02 mole of I_2

IV. Equivalent weight of KIO_3 is $\frac{\text{molar mass}}{5}$

(1) I, IV (2) II, III

(3) II, IV (4) I, III

Ans. **(1)**

Sol. $5I^- + \underset{\text{V.f.} = 5}{IO_3^-} + 6H^+ \rightarrow 3I_2 + 3H_2O$

Moles of KIO_3 for 200 ml KI

$= \frac{0.1 \times 200}{1000} \times \frac{1}{5} = 0.004$

Equivalent of $KIO_3 = \frac{\text{molar mass}}{5}$

10. Match the following.

a.	Apollo aircraft	(i)	Anode Zn, Cathode Carbon rod
b.	Invertor cell	(ii)	Anode Pb, Cathode PbO_2
c.	Transistor	(iii)	Anode Zn/Hg, Cathode HgO
d.	Hearing aid	(iv)	Hydrogen Fuel cell

	a	b	c	d
(1)	(iv)	(ii)	(i)	(iii)
(2)	(i)	(ii)	(iii)	(iv)
(3)	(ii)	(iii)	(iv)	(i)
(4)	(i)	(ii)	(iv)	(iii)

Ans. **(1)**

Sol. (a) Apollo aircraft → Hydrogen Fuel cell

(b) Invertor cell → Anode (Pb), Cathode (PbO_2)

(c) Transistor → Anode (Zn), Cathode (Carbon rod)

(d) Hearing aid → Anode (Zn/Hg), Cathode (HgO)

11. The maximum number of spectral lines observed when electron jumps from the 4^{th} excited state to the ground state of hydrogen atoms?

(1) 6 (2) 3

(3) 10 (4) 2

Ans. **(3)**

Sol. $\dfrac{(n_2 - n_1)(n_2 - n_1 + 1)}{2}$

$= \dfrac{(5-1)(5-1+1)}{2}$

$= \dfrac{20}{2}$

$= 10$

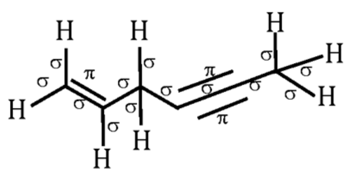
12. Number of σ and π bond respectively in hex-1-en-4-yne are

(1) 13, 3 (2) 14, 3

(3) 3, 14 (4) 14, 13

Ans. **(1)**

Sol.



Total number of σ-bond = 13

Total number of π-bond = 3

13. Which of the following form most stable carbocation?

(1) $(Ph)_3C$-Br

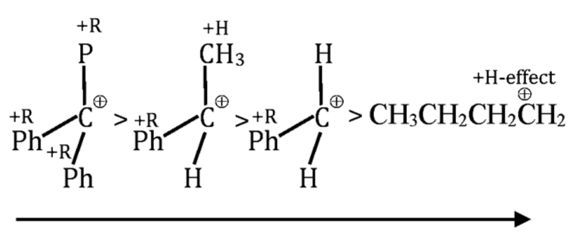
(2) $C_6H_5CH_2Br$

(3) $C_6H_5CH(Br)CH_3$

(4) $CH_3CH_2CH_2CH_2Br$

Ans. **(1)**

Sol.



Carbocation stability order

14. Which of the following are essential amino acid.

(a) Valine (b) Proline

(c) Threonine (d) Tyrosine

(e) Methionine

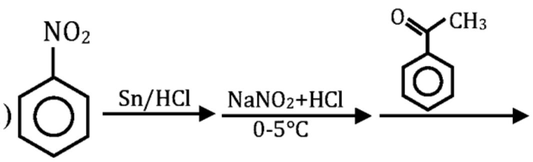
(1) a & b (2) a, c & e

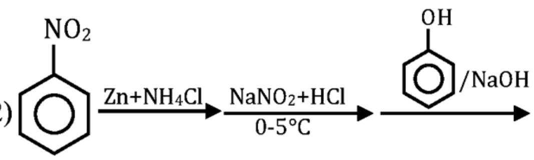
(3) a, b & d (4) b & d

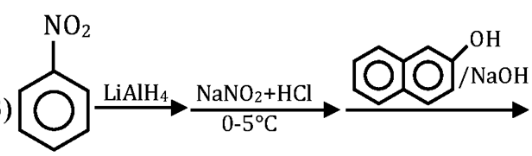
Ans. **(2)**

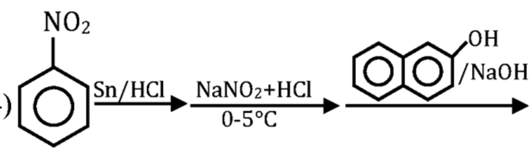
Sol. Valine, Threonine and Methionine are essential amino acid.

15. In which of the following reaction sequence azo dye is formed.

(1) 

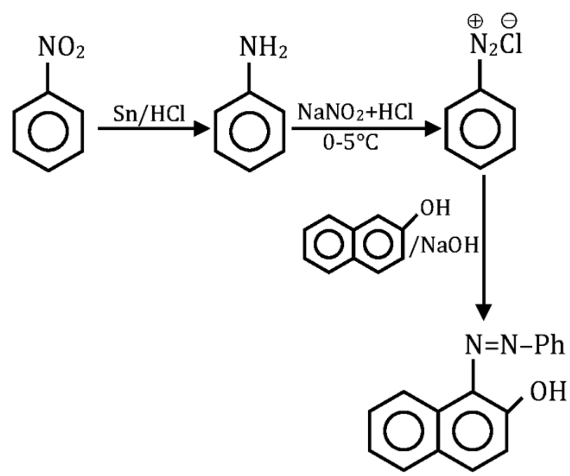
(2) 

(3) 

(4) 

Ans. **(4)**

Sol.



(Orange red dye)

16. **Statement-1:** In paper chromatography stationary phase is paper.

Statement-2: In thin layer chromatography liquid used as stationary phase.

(1) Statement 1 and statement 2, both are correct.

(2) Statement 1 is correct and statement 2 is incorrect.

(3) Statement 1 is incorrect and statement 2 is correct.

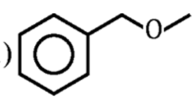
(4) Statement 1 and statement 2, both are incorrect.

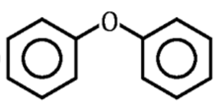
Ans. **(4)**

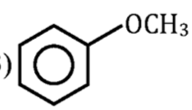
Sol. In paper chromatography stationary phase is water trapped in paper.

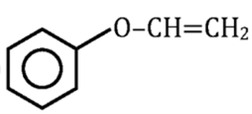
In thin layer chromatography solid substance like Silica gel and alumina are used stationary phase and liquid is used as mobile phase.

17. Which of the following compound will give phenol on reaction with HBr?

(1) 

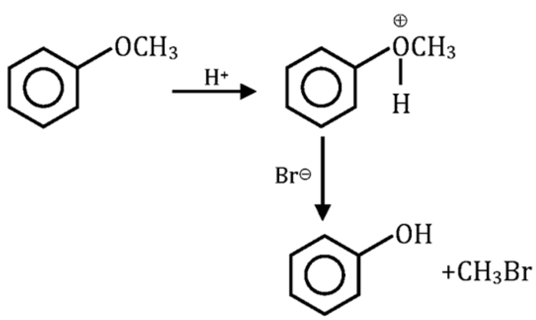
(2) 

(3) 

(4) 

Ans. **(3)**

Sol.



18. Statement-1: meta-xylene on nitration followed by oxidation gives 4-nitrobenzene 1,3-dicarboxylic acid.

Statement-2: Methyl group is ortho/para character while nitro group is meta character.

(1) Both Statements are correct.

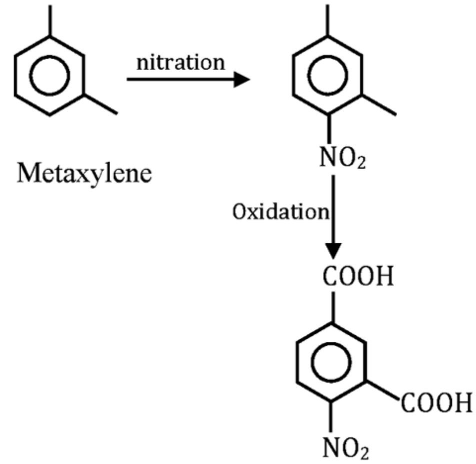
(2) Both Statements are incorrect.

(3) Statement 1 is incorrect and statement 2 is correct.

(4) Statement 1 is correct and statement 2 is incorrect.

Ans. **(1)**

Sol. (i)



(ii) Methyl group → ortho/para directing

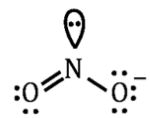
Nitro group → meta directing

SECTION-B

1. Total number of non-bonding electrons in NO_2^-.

Ans. **(12)**

Sol. NO_2^-



Total number of nonbonding electron = 12

2. Find spin only magnetic moment of yellow coloured complex compound

$K_3[Co(NO_2)_6]$, $Cu_2[Fe(CN)_6]$, $Zn_2[Fe(CN)_6]$, $Cu_3[Fe(CN)_6]_2$.

Ans. **(0)**

Sol. $K_3[Co(NO_2)_6]$ → Yellow

$Cu_2[Fe(CN)_6]$ → Chocolate Brown

$Zn_2[Fe(CN)_6]$ → White or bluish-white

$Cu_3[Fe(CN)_6]_2$ → Greenish brown

$K_3[Co(NO_2)_6] \rightarrow d^2sp^3$

No. of unpaired electron = 0

3. 0.41 g of $BaSO_4$ is obtained from 0.2 g of organic compound in carius method. What is percentage of sulphur present in organic compound? (Give your answer to the nearest integer)

Ans. **(28)**

Sol.

$$\%S = \frac{W_{BaSO_4}}{(M_{wt})_{BaSO_4}} \times \frac{32}{W_{total}} \times 100$$

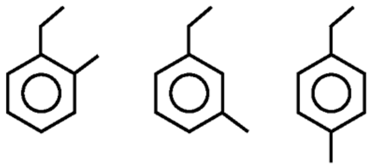
$$\%S = \frac{0.41}{233} \times \frac{32}{0.2} \times 100$$

$\%S = 28.154$

4. A compound having molecular formula C_9H_{12} does not give Baeyer's test. Possible structural isomers of C_9H_{12} which have four non aliphatic substitution sites are-

Ans. **(3)**

Sol. DU of $C_9H_{12} = 4$



5. Using Claisen Schmidt reaction 4.5 g dibenzalacetone is formed using 5.3 gm of benzaldehyde. Find the percentage yield of dibenzalacetone. (Give your answer to the nearest integer)

Ans. **(77)**

Sol.

$$2\ Ph\text{–}CH{=}O + CH_3\text{–}\underset{\underset{O}{\|}}{C}\text{–}CH_3 \xrightarrow{\text{Claisen Schmidt reaction}} Ph\text{–}CH{=}CH\text{–}\underset{\underset{O}{\|}}{C}\text{–}CH{=}CH\text{–}Ph$$

5.3gm $\simeq \frac{1}{20}$ moles

4.5gm

Expected yield (100%) of dibenzalacetone from $\frac{1}{20}$ moles of benzaldehyde $= \frac{1}{40}$ moles

(i.e. 5.85gm)

%yield of dibenzalacetone

$$= \frac{4.5}{5.85} \times 100 = 76.92 \simeq 77$$

SECTION-A

1. If the letters of the word "KANPUR" are arranged in dictionary, then 440th word is

(1) PRKAUN (2) PRKUAN

(3) PRKNAU (4) PRKUNA

Ans. **(1)**

Sol. Letters starting with

A _ _ _ _ _ = 5! = 120

K _ _ _ _ _ = 5! = 120

N _ _ _ _ _ = 5! = 120

P A _ _ _ _ = 4! = 24

P K _ _ _ _ = 4! = 24

P N _ _ _ _ = 4! = 24

P R A _ _ _ = 3! = 6

P R K A N U → 439th

P R K A U N → 440th

2. If 7^{103} is divided by 23, the remainder is

Sol. By FLT 23 is a prime number

$7^{22} \equiv 1 \pmod{23}$

$\Rightarrow 7^{103} = 7^{88+15} = 7^{88}.7^{15}$

$\Rightarrow 7^{103} \equiv 7^{15} \pmod{23}$

So, $(343)^5$

$7^{15} = (343)^5 = (345 - 2)^5$

$\Rightarrow 7^{15} = 23\lambda_1 + (-32)$

$= 23\lambda_2 - 32 + 96$

$= 23\lambda_2 + 14$

So, $7^{103} = 23\mu + 14$

3. Let $a_{ij} = (\sqrt{2})^{i+j}, A = [a_{ij}]_{3\times3}$. If sum of third row of A^2 is $\alpha + \beta\sqrt{2}$, then $\alpha + \beta$ is

Ans. **(224)**

Sol. $A = \begin{bmatrix} 2 & 2\sqrt{2} & 4 \\ 2\sqrt{2} & 4 & 4\sqrt{2} \\ 4 & 4\sqrt{2} & 8 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} - & - & - \\ - & - & - \\ 56 & 56\sqrt{2} & 112 \end{bmatrix}$

$\alpha + \beta\sqrt{2} = 168 + 56\sqrt{2}$

$\Rightarrow \alpha + \beta = 224$

4. Let $f(x) = \int_0^x t(t^2 - 3t + 20)dt, x \in [1,3]$ and range of f(x) is $[\alpha, \beta]$, then $\alpha + \beta$ is equal to

Ans. $\left(\frac{185}{2}\right)$

Sol. $f'(x) = x(x^2 - 3x + 20) = x^3 - 3x^2 + 20x \uparrow x \in (1, 3)$

$\downarrow$

$D < 0$

$f(1) = \int_0^1 (t^3 - 3t^2 + 20t)dt = \left(\frac{t^4}{4} - \frac{3t^3}{3} + \frac{20t^2}{2}\right)_0^1$

$\Rightarrow f(1) = \frac{1}{4} - 1 + 10 = \frac{37}{4} = \alpha$

$\Rightarrow f(3) = \int_0^3 (t^3 - 3t^2 + 20t)dt = \left(\frac{t^4}{4} - \frac{3t^3}{3} + \frac{20t^2}{2}\right)_0^3$

$= \frac{81}{4} - 27 + 10 \times 9 = \frac{333}{4} = \beta$

$\Rightarrow \alpha + \beta = \frac{37 + 333}{4} = \frac{370}{4} = \frac{185}{2}$

5. The value of the limit

$\lim_{x\to0}(\text{cosec}x)\left(\sqrt{2\cos^2x + 3\cos x} - \sqrt{\cos^2x + \sin x + 4}\right)$

is

(1) 0 (2) 1

(3) $\frac{1}{2\sqrt{5}}$ (4) $-\frac{1}{2\sqrt{5}}$

Ans. **(4)**

Sol. $\lim_{x\to 0}\frac{2\cos^2 x+3\cos x-\left(\cos^2 x+\sin x+4\right)}{\sin x\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$

$$\lim_{x\to 0}\left(\frac{\cos^2 x+3\cos x-4-\sin x}{\sin x}\right)\times\frac{1}{2\sqrt{5}}$$

$$=\frac{1}{2\sqrt{5}}\lim_{x\to 0}\left(\frac{(\cos x+4)(\cos x-1)-\sin x}{\sin x}\right)$$

$$=-\frac{1}{2\sqrt{5}}\lim_{x\to 0}\left(\frac{(\cos x+4)\times 2\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$$=-\frac{1}{2\sqrt{5}}\lim_{x\to 0}\left(\frac{(\cos x+4)+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)=\frac{-1}{2\sqrt{5}}$$

6. $a_1, a_2, a_3, \ldots, a_{2024}$ are in A.P.

If $a_1+(a_5+a_{10}+a_{15}+\cdots+a_{2020})+a_{2024}=2233$, then find the value of $a_1+a_2+\cdots+a_{2024}$

Ans. (4)

Sol. $a_1+a_5+a_{10}+\ldots..+a_{2020}+a_{2024}$

$\Rightarrow 203(a_1+a_{2024})=2233$

$\Rightarrow a_1+a_{2024}=11$

$$S_{2024}=\frac{2024}{2}(a_1+a_{2024})$$

$\Rightarrow S_{2024}=(101)^2\times 11=11132$

7. Let the line L be $\frac{x-1}{1}=\frac{y-4}{3}=\frac{z-7}{5}$ and foot of perpendicular from $(1,-2,-1)$ to L is (α,β,γ), then the value of $\alpha+\beta+\gamma$ is

(1) $-\frac{69}{35}$ (2) $\frac{102}{35}$

(3) $\frac{69}{35}$ (4) $-\frac{102}{35}$

Ans. (4)

Sol. Any point on Lis $P(\lambda+1, 3\lambda+4, 5\lambda+7)$

Given $Q(1,-2,-1)$

Dr's of PQ = $<\lambda, 3\lambda+6, 5\lambda+8>$

$\overrightarrow{PQ}$ is Perpendicular to line L,

So, $\lambda\times 1+(3\lambda+6)\times 3+(5\lambda+8)\times 5=0$

$$\Rightarrow \lambda=\frac{-58}{35}$$

Now sum of Coordinates of P is

$$\Rightarrow \lambda+1+3\lambda+4+5\lambda+7=9\lambda+12=\frac{-102}{35}$$

8. If the exhaustive values of a for which the equation $2x^2+(a-5)x+15=3a$ has no real roots is (α,β), then $|4(\alpha+\beta)|$ is equal to

(1) 56 (2) 52

(3) 54 (4) 18

Ans. (1)

Sol. $2x^2+(a-5)x+15=3a$

$\Rightarrow 2x^2+(a-5)x+(15-3a)=0$

$D<0$

$(a-5)^2-8(15-3a)<0$

$\Rightarrow a^2+14a-95<0$

$\Rightarrow (a+19)(a-5)<0$

$a\in(-19, 5)$

$\therefore \alpha=-19$

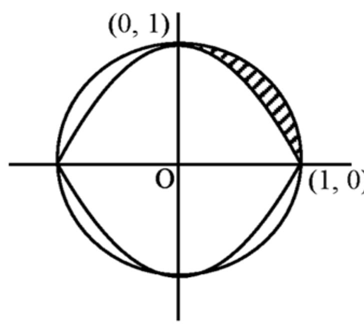
$\beta=5$

$\therefore \alpha+\beta=-14$

$\therefore |4(\alpha+\beta)|=56$

9. Let the area bounded by the curves $|y|=1-x^2$, $x^2+y^2=1$ is α. If $3\alpha=\beta\pi+\gamma$, then find $|\beta-\gamma|$

Sol.



$$\alpha=A=4\left[\frac{\pi}{4}-\int_0^1\left(1-x^2\right)dx\right]$$

$$\Rightarrow \alpha=4\left[\frac{\pi}{4}-\frac{2}{3}\right]$$

$$\Rightarrow \alpha = \pi - \frac{8}{3} \Rightarrow 3\alpha = 3\pi - 8$$

$$\beta = 3, \gamma = -8$$

$$|\beta - \gamma| = 11$$

10. If $x + y + z = 1,\ x + 2y + 4z = m\ \&\ x + 4y + 10z = m^2$ have infinite solutions and m takes 2 values α & β, then find

$$\sum_{r=1}^{10} (r)^{\alpha} + (r)^{\beta}$$

Ans. **(440)**

Sol. For infinite solutions $D = D_1 = D_2 = D_3 = 0$

So, $D_1 = \begin{vmatrix} 1 & 1 & 1 \\ m & 2 & 4 \\ m^2 & 4 & 10 \end{vmatrix} = 0 \Rightarrow (m^2 - 3m + 2) = 0$

$$\sum_{r=1}^{10} (r + r^2) = \sum_{r=1}^{10} r + \sum_{r=1}^{10} r^2$$

$$\Rightarrow \frac{10 \times 11}{2} + \frac{10 \times 11 \times 21}{6}$$

$= 440$

11. If $\log y = x \log \frac{2}{5}, x \in \mathbb{N} \cup \{0\}$. Then sum of all values of y is equals to

(1) $\frac{5}{3}$ (2) $\frac{2}{3}$

(3) $\frac{5}{4}$ (4) $\frac{8}{3}$

Ans. **(1)**

Sol. $\left(\log \frac{2}{5}\right) x = \log y$

$x = 0, y = 1$

$x = 1, y = \frac{2}{5}$

$x = 2,\ y = \left(\frac{2}{5}\right)^2$

⋮ ⋮

∞ ∞

so, $\sum y = \frac{1}{1 - \frac{2}{5}} = \frac{5}{3}$

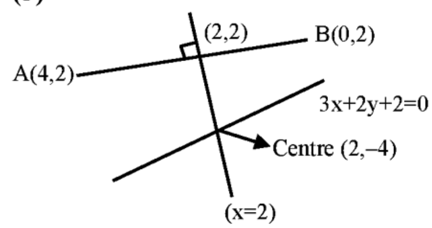
12. Two points (4 ,2) and (0, 2) lie on the circle whose centre lies on $3x + 2y + 2 = 0$, then length of the chord whose mid-point is (1, 2), is

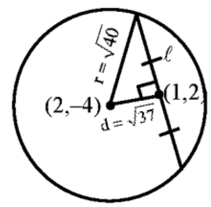
(1) $\sqrt{3}$ (2) $\sqrt{5}$

(3) $2\sqrt{3}$ (4) $2\sqrt{5}$

Ans. **(3)**

Sol.





$r = \sqrt{40} = 2\sqrt{10}$

$\ell = 2\sqrt{r^2 - d^2} = 2\sqrt{40 - 37} = 2\sqrt{3}$

13. The value of $\int_0^{\frac{\pi}{4}} \left(\sin\left|\left(4x - \frac{\pi}{2}\right)\right| + \sin[2x]\right) dx$ is

(where [.] denotes the greatest integer function)

(1) $\frac{1}{2} + \left(\frac{\pi - 2}{4}\right)\sin 1$ (2) $\frac{1}{4} + \left(\frac{\pi - 2}{2}\right)\sin 1$

(3) $\frac{1}{2} - \left(\frac{\pi - 2}{4}\right)\sin 1$ (4) $\frac{1}{4} - \left(\frac{\pi - 2}{2}\right)\sin 1$

Ans. **(1)**

Sol. $\int_0^{\frac{\pi}{4}} \left(\sin\left|4x - \frac{\pi}{2}\right| + \sin[2x]\right) dx$

$=$

$$\int_0^{\frac{\pi}{8}} -\sin\left(4x - \frac{\pi}{2}\right) dx + \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin\left(4x - \frac{\pi}{2}\right) dx + \int_0^{\frac{1}{2}} 0\, dx + \int_{\frac{1}{2}}^{\frac{\pi}{4}} \sin 1\, dx$$

$$= \left.\frac{\cos\left(4x - \frac{\pi}{2}\right)}{4}\right|_0^{\frac{\pi}{8}} - \left.\frac{\cos\left(4x - \frac{\pi}{2}\right)}{4}\right|_{\frac{\pi}{8}}^{\frac{\pi}{4}} + \left(\frac{\pi}{4} - \frac{1}{2}\right)\sin 1$$

$$= \left(\frac{1}{4} - 0\right) - \left(0 - \frac{1}{4}\right) + \left(\frac{\pi - 2}{4}\right)\sin 1$$

$$= \frac{1}{2} + \frac{(\pi - 2)\sin 1}{4}$$

14. If the domain of $log_{x-1}\left(\frac{2x^2-9x+4}{x^2-4x+5}\right)$ is (α,∞) and $log_5(18x-x^2-77)$ is (β,γ), then the value of $\alpha^2+\beta^2+\gamma^2$ is

Sol. $x - 1 > 0 \Rightarrow x > 1$

$\& \ \frac{2x^2-9x+4}{x^2-4x+5} > 0$

$x^2 - 4x + 5 > 0 \to D > 0$

and $2x^2 - 9x + 4 > 0$

$\Rightarrow (x-4)\left(x-\frac{1}{2}\right) > 0$

$\Rightarrow x \in (4, \infty) = (\alpha, \infty) \Rightarrow \alpha = 4$

and $18x - x^2 - 77 > 0$

$\Rightarrow x^2 - 18x + 77 < 0$

$\Rightarrow (x - 11)(x - 7) < 0$

$\Rightarrow x \in (7, 11) = (\beta, \gamma) \Rightarrow \beta = 7, \gamma = 11$

$\alpha^2 + \beta^2 + \gamma^2 = 16 + 49 + 121 = 186$

15. If $sinx + sin^2 x = 1$, then find the value of $cos^{12} x + tan^{12} x + 3[cos^8 x + tan^8 x] + cos^6 x + tan^6 x$

Sol. $\sin x = \cos^2 x \Rightarrow \sin^2 x + \sin x - 1 = 0$

$\cos^{12}x + \tan^{12}x + 3[\cos^8 x + \tan^8 x] + \cos^6 x + \tan^6 x$

Now,

$$\begin{array}{r|l} & 2\sin^6 x + 6\sin^4 x + 2\sin^3 x \\ \sin^2 x + \sin x - 1 & 2\sin^6 x + 6\sin^4 x + 2\sin^3 x \\ & \underline{2\sin^6 x + 2\sin^5 x - 2\sin^4 x} \\ & -2\sin^5 x + 8\sin^4 x + 2\sin^3 x \\ & \underline{-2\sin^5 x - 2\sin^4 x + 2\sin^3 x} \\ & 10\sin^4 x \end{array}$$

$\because \sin^2 x + \sin x - 1 = 0$

$\Rightarrow \sin x = \frac{-1+\sqrt{5}}{2}$

$\Rightarrow \sin^2 x = \frac{3-\sqrt{5}}{2}$

$\Rightarrow \sin^4 x = \left(\frac{7-3\sqrt{5}}{2}\right)$

$\Rightarrow 10\sin^4 x = 35 - 15\sqrt{5}$

16. Bag 1 has 4 white and 5 black balls and bag 2 has n white and 3 black balls. A ball is chosen at random from bag 1 and put into bag 2, then a ball is drawn from bag 2. If probability of getting a white ball from bag 2 is $\frac{29}{45}$, then find the value of n

Sol. B_1 {4 white, 5 Black}

B_2 {n white, 3 Black}

$P\left(\frac{W}{B_2}\right) = \frac{29}{45}$

$\Rightarrow$ P(white ball from bag 1) × P(White ball from Bag 2) + P(Black ball from bag 1) × P(White ball from bag 2) = $\frac{29}{45}$

$\Rightarrow \frac{4}{9} \times \frac{n+1}{n+4} + \frac{5}{9} \times \frac{n}{n+4} = \frac{29}{45}$

$\Rightarrow \frac{45}{9}(4n + 4 + 5n) = 29(n + 4)$

$\Rightarrow 5(9n + 4) = 29n + 116$

$16n = 116 - 20$

$n = 6$

17. If $\lim_{t\to 0}\left(\int_0^1 (3x+5)^t dx\right)^{\frac{1}{t}} = \frac{\alpha}{5e}\cdot\left(\frac{8}{5}\right)^{\frac{2}{3}}$, then find the value of α

Ans. **(64)**

Sol. $\frac{\alpha}{5e} = \exp\left(\lim_{t\to 0}\frac{1}{t}\left(\int_0^1 (3x+5)^t dx - 1\right)\right)$

$= \exp\left(\lim_{t\to 0}\frac{1}{t}\left(\left.\frac{(3x+5)^{t+1}}{3(t+1)}\right|_0^1 - 1\right)\right)$

$= \exp\left(\lim_{t\to 0}\left(\frac{1}{t}\left(\frac{8^{t+1}-5^{t+1}}{3(t+1)} - 1\right)\right)\right)$

$= \exp\left(\lim_{t\to 0}\left(\frac{1}{t}\cdot\frac{8^{t+1}-5^{t+1}-3t-3}{3(t+1)}\right)\right)$

$$= \exp\left(\lim_{t\to 0}\left(\frac{8^{t+1}\cdot \ln 8 - 5^{t+1}\ln 5 - 3}{3(t+1)}\right)\right)$$

$$= \exp\left(\frac{\ln 8^8 - \ln 5^5 - 3}{3}\right)$$

$$\Rightarrow \left(\frac{8}{3}\right)^{\frac{2}{3}} \frac{\alpha}{5e} = \exp\left(\frac{\ln\left(\frac{8^8}{5^5}\right)}{3} - 1\right)$$

$$\Rightarrow \left(\frac{8}{3}\right)^{\frac{2}{3}} \frac{\alpha}{5} = \left(\frac{8^8}{5^5}\right)^{\frac{1}{3}} = \left(\frac{8^6\cdot 8^2}{5^3\cdot 5^2}\right)^{1/3} = \frac{64}{5}\left(\frac{8}{5}\right)^{\frac{2}{3}}$$

$$\Rightarrow \alpha = 64$$

18. If $\frac{dy}{dx} + (\tan x)y = \frac{2+\sec x}{(1+2\sec x)^2}$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{10}$, then find the value of $f\left(\frac{\pi}{4}\right)$

Sol. $\text{I.F.} = e^{\int \tan x dx}$

$$= e^{\ln \sec x} = \sec x$$

$$\Rightarrow \sec x \frac{dy}{dx} + \sec x.\tan x.y = \frac{2+\sec x}{(1+2\sec x)^2}\sec x$$

$$\Rightarrow \sec x\, y = \int \frac{2\sec x + \sec^2 x}{(1+2\sec x)^2} dx$$

$$= \int\left(\frac{2\sec x + \sec^2 x}{\sec x \tan x}\right).\frac{\sec x \tan x}{(1+2\sec x)^2} dx$$

$$\int (2\cot x + \operatorname{cosec} x)\frac{-1}{2(1+2\sec x)} dx$$

$$\Rightarrow \frac{-(2\cot x + \operatorname{cosec} x)}{2(1+2\sec x)} - \int \frac{1}{\sin^2 x}.\frac{2+\cos x}{2(\cos x + 2)}.\cos x dx$$

$$\sec x.y = \frac{-(2\cot x + \operatorname{cosec} x)}{2(1+2\sec x)} + \frac{1}{2}\operatorname{cosec} x + c$$

$$x = \frac{\pi}{3}$$

$$2.\frac{\sqrt{3}}{10} = \frac{-\left(2.\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}\right)}{2(1+2.2)} + \frac{1}{2}.\frac{2}{\sqrt{3}} + c$$

$$\Rightarrow \frac{\sqrt{3}}{5} = \frac{-4}{\sqrt{3}}.\frac{1}{10} + \frac{1}{\sqrt{3}} + c$$

$$\Rightarrow c = \frac{\sqrt{3}}{5} + \frac{2}{5\sqrt{3}} - \frac{1}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{5} - \frac{\sqrt{3}}{5} = 0$$

19. The equation $\alpha x + \beta y = 109$ is a chord of ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ having midpoint $\left(\frac{5}{2}, \frac{1}{2}\right)$, then $\alpha + \beta$ is equal to

Ans. **(58)**

Sol. $T = S_1$

$$\Rightarrow \frac{5}{2}.\frac{x}{9} + \frac{y}{4}.\frac{1}{2} - 1 = \frac{\left(\frac{5}{2}\right)^2}{9} + \frac{\left(\frac{1}{2}\right)^2}{4} - 1$$

$$\Rightarrow \frac{5x}{18} + \frac{y}{8} - 1 = \frac{25}{36} + \frac{1}{16} - 1$$

$$\Rightarrow \frac{5x}{9} + \frac{y}{4} = \frac{25}{18} + \frac{1}{8}$$

$$\Rightarrow 20x + 9y = 50 + \frac{9}{2}$$

$$\Rightarrow 40x + 18y = 109$$

$$\Rightarrow \alpha + \beta = 58$$