

बेतियाहाता चौक पर पिछले 21 वर्षों से संचालित पूर्वांचल की No.1 कोचिंग

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MOMENTUM

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Head Office

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IIT-JEE

NEET (UG)

Foundations

Memory Based Answers & Solutions

Time : 3 hrs.

for

M.M. : 300

JEE (Main)-2025 (Online) Phase-1

(Physics, Chemistry and Mathematics)

29 Jan 2025 (Morning Shift)

IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.
- (3) This question paper contains **Three Parts**. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is **Mathematics**. Each part has only two sections: **Section-A** and **Section-B**.
- (4) **Section - A** : Attempt all questions.
- (5) **Section - B** : Attempt all questions.
- (6) **Section - A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **-1 mark** for wrong answer.
- (7) **Section - B (21 – 25)** contains 5 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **-1 mark** for wrong answer.

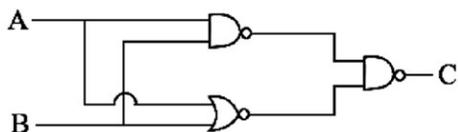
5. Two projectiles were launched from same position simultaneously with same speed. One of the projectile was launched at angle $(45 - \alpha)^\circ$ and the other at an angle of $(45 + \alpha)^\circ$ with the horizontal. Find the ratio of maximum height of the projectiles.

- (1) $\frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}$ (2) $\frac{1 - \sin 2\alpha}{1 + \sin 2\alpha}$
 (3) $\frac{1 - \tan \alpha}{1 + \tan \alpha}$ (4) $\frac{1 - \sin \alpha}{1 + \sin \alpha}$

Ans. (2)

Sol. $h_{\max} = \frac{u^2 \sin^2 \theta}{2g}$
 $\frac{h_1}{h_2} = \frac{\sin^2(45 - \alpha)}{\sin^2(45 + \alpha)} = \frac{1 - \cos(90 - 2\alpha)}{1 - \cos(90 + 2\alpha)}$
 $= \frac{1 - \sin 2\alpha}{1 + \sin 2\alpha}$

6. Identify the logic gate represented by the circuit shown below.



- (1) NOR Gate (2) NAND Gate
 (3) AND Gate (4) OR Gate

Ans. (4)

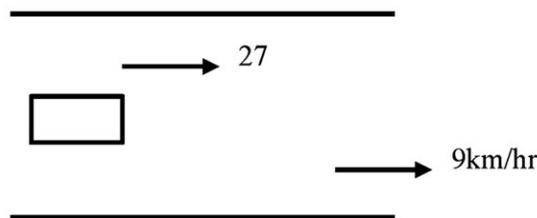
Sol. $\overline{(A \cdot B)} \cdot \overline{(A + B)}$
 $\overline{(\overline{A \cdot B})} + \overline{(\overline{A + B})}$
 $A \cdot B + A + B$
 $A(B + 1) + B \Rightarrow A + B \Rightarrow \text{OR Gate}$

7. A river is flowing with speed 9 km/h. Boat is going downstream-speed of boat in still water is 27 km/h. A person in boat throws a ball upwards with speed 10 m/s. Find range of the ball as seen by an observer at bank of river

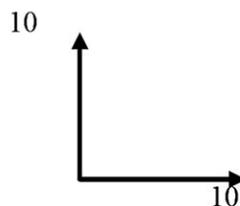
- (1) $20\sqrt{3}$ m (2) 20 m
 (3) 30 m (4) $10\sqrt{3}$ m

Ans. (2)

Sol.



$$36 \times \frac{15}{18} = 10 \text{ m/s}$$



$$R = \frac{2u_y u_x}{g} = \frac{2 \times 10 \times 10}{10} = 20 \text{ m}$$

8. Statement-I : Electromagnetic wave have both energy and momentum.

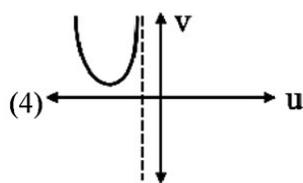
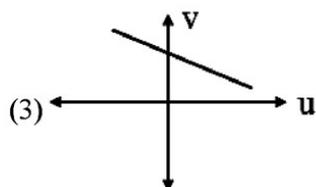
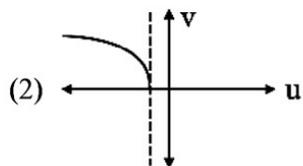
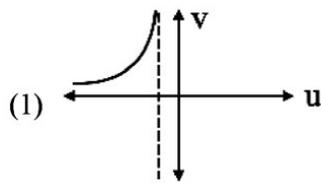
Statement-II : Rest mass of photon is zero.

- (1) Statement-I is correct, statement-II is correct and statement-II is correct explanation of statement-I.
 (2) Statement-I is correct, statement-II is correct and statement-II is not the correct explanation of statement-I.
 (3) Statement-I is correct and statement-II is incorrect.
 (4) Statement-I is incorrect and statement-II is correct.

Ans. (2)

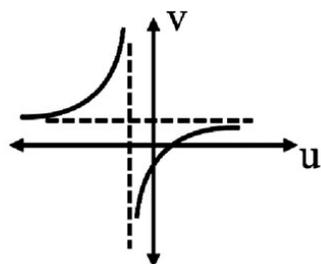
Sol. Theoretical $E = \frac{hc}{\lambda}$; $P = \frac{h}{\lambda}$

9. For a convex lens having $|u| > f$, where f is focal length. Graph between v & u is :



Ans. (1)

Sol.



10. Which of two physical quantities have same dimensions ?

- (1) Angular momentum and Planck's constant
- (2) Torque and moment of inertia
- (3) Impulse and surface tension
- (4) Momentum and work done

Ans. (1)

Sol. $L = mvr = [M] [LT^{-1}] [L] = [M^1L^2T^{-1}]$

$E = hf$

$[M^1L^2T^{-2}] = [h] [T^{-1}]$

$[h] = [M^1L^2T^{-1}]$

11. If radius of first Bohr's orbit of H-atom is a_0 . Then find the radius of 2nd Bohr's orbit of H-atom.

- (1) $8a_0$
- (2) $4a_0$
- (3) a_0
- (4) $6\pi a_0$

Ans. (2)

Sol. $r = a_0 \frac{n^2}{z}$

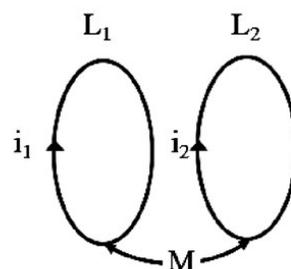
$n = 1, z = 1$

$r = a_0$

$n = 2, z = 2$

$r = 4a_0$

12. Two coils having self-inductance L_1 and L_2 are placed closely such that they have a mutual inductance M . If they carry currents i_1 and i_2 as shown in the figure, then the induced emf in coil 1 is



(1) $-L_1 \left(\frac{di_1}{dt} \right) + M \left(\frac{di_2}{dt} \right)$

(2) $-L_1 \left(\frac{di_1}{dt} \right) - M \left(\frac{di_2}{dt} \right)$

(3) $-L_1 \left(\frac{di_2}{dt} \right) + M \left(\frac{di_1}{dt} \right)$

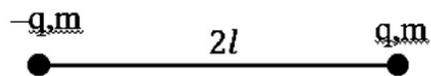
(4) $-L_1 \left(\frac{di_2}{dt} \right) - M \left(\frac{di_1}{dt} \right)$

Ans. (2)

Sol. $\phi_1 = L_1 i_1 + M i_2$ (same direction of current)

$$\varepsilon_1 = \frac{-d\phi_1}{dt} = \frac{-L di_1}{dt} - \frac{M di_2}{dt}$$

13. An electric dipole is placed in uniform electric field 'E' in stable equilibrium. Find time period of small oscillations on disturbance.



(1) $T = 2\pi \sqrt{\frac{ml}{qE}}$

(2) $T = 2\pi \sqrt{\frac{2ml}{qE}}$

(3) $T = \frac{1}{2\pi} \sqrt{\frac{ml}{qE}}$

(4) $T = \frac{1}{2\pi} \sqrt{\frac{2ml}{qE}}$

Ans. (1)

Sol. $\vec{\tau} = \vec{P} \times \vec{E}$

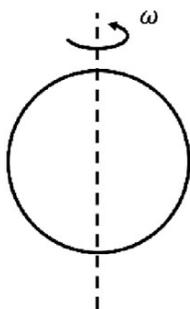
$$\alpha = \frac{PE}{I} \theta$$

$$T = 2\pi \sqrt{\frac{I}{PE}}$$

$$T = 2\pi \sqrt{\frac{2m\ell^2}{q(2\ell)E}}$$

$$T = 2\pi \sqrt{\frac{m\ell}{qE}}$$

14. A coil of area 'A' and number of turns 'N' is rotating about diameter in uniform magnetic field B. Find induced emf and flux when magnetic field is parallel to plane of coil



- (1) $-NBA\omega, 0$ (2) $0, 0$
 (3) $2NBA\omega, NBA$ (4) $-NBA\omega, 2NBA$

Ans. (1)

Sol. $\phi = BAN \sin(\omega t)$

$$\varepsilon = -\frac{d\phi}{dt} = -BAN\omega \cos(\omega t)$$

At $t = 0, \phi = 0$

$$\varepsilon = -NBA\omega$$

15. Statement-I : A negative potential is required to stop the photoelectrons.

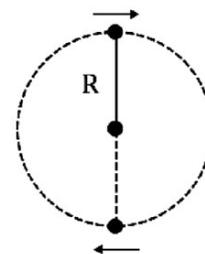
Statement-II : Speed of electron decreases when a negative potential is applied in a photo cell.

- (1) Statement-I is correct, statement-II is correct and statement-II is correct explanation of statement-I.
 (2) Statement-I is correct, statement-II is correct and statement-II is not the correct explanation of statement-I.
 (3) Statement-I is correct and statement-II is incorrect.
 (4) Statement-I is incorrect and statement-II is correct.

Ans. (1)

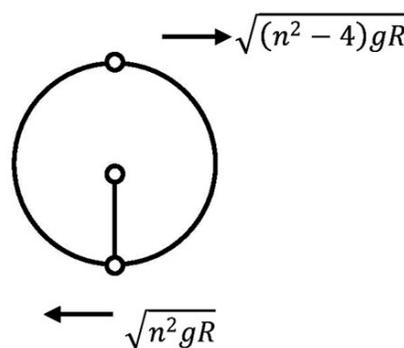
Sol. Theoretical

16. The particle is vertical circular motion with the help of a string and has speed $n\sqrt{gR}$. Find ratio of kinetic energy at top and bottom



- (1) $\frac{KE_{top}}{KE_{bottom}} = \frac{n^2 - 4}{n^2}$ (2) $\frac{KE_{top}}{KE_{bottom}} = \frac{n^2 + 4}{n^2}$
 (3) $\frac{KE_{top}}{KE_{bottom}} = \frac{n^2 - 2}{n^2}$ (4) $\frac{KE_{top}}{KE_{bottom}} = \frac{n^2 + 2}{n^2}$

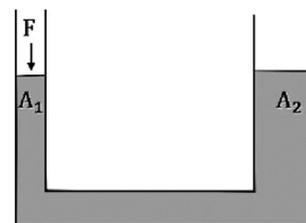
Ans. (1)



Sol.

$$\frac{KE_{top}}{KE_{bottom}} = \frac{n^2 - 4}{n^2}$$

17. If applied force causes rise in wider piston is x Find work done by force F.



- (1) $\frac{A_2}{A_1} Fx$ (2) $\frac{A_1}{A_2} Fx$
 (3) $\frac{A_2}{A_1 Fx}$ (4) $\frac{A_1}{A_2 Fx}$

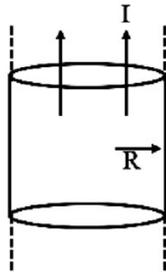
Ans. (1)

Sol. $A_1 x_1 = A_2 x$

$$x = \frac{A_2}{A_1} x$$

$$\therefore W = f \cdot x_1 = \frac{A_2}{A_1} Fx$$

18. An infinite solid cylindrical wire of radius R carries a current I uniformly distributed along its area. The distance from the centre where the magnetic field is equal to $\frac{\mu_0 I}{4\pi R}$ is



- (1) $\frac{R}{2}$ (2) R
 (3) $4R$ (4) Zero

Ans. (1)

Sol. $\frac{\mu_0 J a^2}{2r} = \frac{\mu_0 J a}{4}$

$R = 2a$

19. Half mole of Argon gas is given heat 500 J in isobaric process. Find rise in temperature and change in internal energy.

- (1) $\Delta T = 78 \text{ K}, \Delta U = 200 \text{ J}$
 (2) $\Delta T = 48 \text{ K}, \Delta U = 300 \text{ J}$
 (3) $\Delta T = 88 \text{ K}, \Delta U = 250 \text{ J}$
 (4) $\Delta T = 68 \text{ K}, \Delta U = 300 \text{ J}$

Ans. (2)

Sol. $\Delta Q = 500 \text{ J} = nC_p \Delta T$

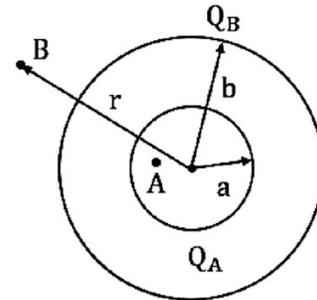
$$\Delta T = \frac{500}{nC_p} = \frac{500}{\frac{1}{2} \times \frac{5R}{2}} = \frac{400}{25/3} \text{ K} = 48 \text{ K}$$

$\Delta U = nC_v \Delta T$

$$= nC_p \Delta T \cdot \frac{C_v}{C_p} = 500 \times \frac{3}{5} = 300 \text{ J}$$

20. There are two spherical shells of radius a & b and charges Q_A & Q_B respectively. There are two points A & B as shown

(E = Electric field, B = Magnetic field). Then



- (1) $E_A = \frac{KQ_A}{a^2}, E_B = 0$
 (2) $E_A = 0, E_B = \frac{K(Q_A+Q_B)}{r^2}$
 (3) $B_A = \frac{\mu_0 \cdot (Q_A+Q_B)}{2\pi a}, E_B = \frac{K(Q_A+Q_B)}{r^2}$
 (4) $B_A = 0, B_B = \frac{\mu_0 \cdot (Q_A+Q_B)}{2\pi r}$

Ans. (2)

Sol. Electric field inside shell is zero, $E_A = 0$

Electric field at outside point B, $E_B = \frac{K(Q_A+Q_B)}{r^2}$

21. In a adiabatic process of closed system, work done by the gas depends explicitly on

- (1) Change in pressure
 (2) Change in volume
 (3) Change in temperature
 (4) Change in number of moles

Ans. (3)

Sol. $Q = W + \Delta U = 0$

$W = -\Delta U$

$= -nC_v \Delta T$

SECTION-B

22. A ball falling in a sea of depth 2.5 km shows $x\%$ decrease in its volume at the bottom. The bulk modulus of material of ball is $20 \times 10^9 \frac{N}{m^2}$. Find '100x'

Ans. (125)

Sol. $B = \frac{\rho gh}{x/100}$

$$x = \frac{\rho gh \times 100}{B} = \frac{1000 \times 10 \times 2500 \times 100}{2 \times 10^9}$$

$$= 1.25$$

23. An ideal gas 27°C is heated to double its pressure at constant volume. Find final temperature is $^\circ\text{C}$.

Ans. (327)

Sol. $PV = nRT$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{P}{300} = \frac{2P}{T}$$

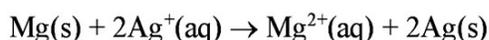
$$T = 600 \text{ K}$$

$$T = 327 \text{ }^\circ\text{C}$$

6. Given : $E_{\text{Ag}^+/\text{Ag}}^\circ$
 $E_{\text{Mg}^{2+}/\text{Mg}}^\circ$
 Which is the correct representation of cell potential of
 $\text{Mg(s)} \mid \text{Mg}^{2+}(\text{aq}) \parallel \text{Ag}^+(\text{aq}) \mid \text{Ag(s)}$
- (1) $E_{\text{cell}}^\circ - \frac{0.059}{2} \log \frac{[\text{Ag}^+]^2}{[\text{Mg}^{2+}]}$
 (2) $E_{\text{cell}}^\circ - \frac{0.059}{2} \log \frac{[\text{Ag}^+]}{[\text{Mg}^{2+}]}$
 (3) $E_{\text{cell}}^\circ - \frac{0.059}{2} \log \frac{[\text{Mg}^{2+}]}{[\text{Ag}^+]^2}$
 (4) $E_{\text{cell}}^\circ - \frac{0.059}{2} \log \frac{[\text{Mg}^{2+}]}{[\text{Ag}^+]}$

Ans. (3)

Sol. Cell reaction is



Nernst equation

$$E_{\text{cell}} = E_{\text{cell}}^\circ - \frac{0.059}{2} \log \frac{[\text{Mg}^{2+}]}{[\text{Ag}^+]^2}$$

7. What is the value of Van't Hoff Factor for A_2B . if the percentage dissociation is 30% ?

- (1) 1.60 (2) 1.30
 (3) 1.50 (4) 1.20

Ans. (1)

Sol. $i = 1 + (n - 1) \alpha$

For A_2B $n = 3$

Given $\alpha = 0.3$

$$i = 1 + (3 - 1) 0.3$$

$$i = 1.6$$

8. $1/2$ mole of Ar gas kept in a closed vessel at 298 K & 1 atm absorbs 500 J heat. Final temperature of gas & change in internal energy is ($R = 8.3$ J/mol-K)

- (1) 346 K, 300 J (2) 346 K, 500 J
 (3) 398 K, 500 J (4) 378 K, 300 J

Ans. (1)

Sol. $q_p = nC_{p,m} \Delta T$

$$500 = \frac{1}{2} \times \frac{5}{2} R \times \Delta T$$

$$\Delta T \approx 48 \text{ K}$$

$$T_f = 48 + 298 = 346$$

$$q_v = \frac{1}{2} \times \frac{3}{2} \times 8.3 \times 48$$

$$q_v = \Delta U = 300 \text{ J}$$

9. Degree of dissociation of $\text{AB}_2(\text{g})$ into A_2 and B_2 in α .

Total equilibrium pressure = P .

Equilibrium constant of the reaction = K_p .

Correct option is — (α is negligible as compared to unity)

- (1) $\left(\frac{2K_p^2}{P}\right)^{\frac{1}{3}}$ (2) $\left(\frac{4K_p^3}{P}\right)^{\frac{1}{3}}$
 (3) $\left(\frac{K_p^2}{2P}\right)^{\frac{2}{3}}$ (4) $\left(\frac{2K_p}{P}\right)^{\frac{1}{3}}$

Ans. (1)

Sol. $\text{AB}_2(\text{g}) \rightarrow \frac{1}{2} \text{A}_2(\text{g}) + \text{B}_2(\text{g})$

$$t = 0 \quad 1 \quad 0 \quad 0$$

$$t = t_{\text{eq}} \quad 1 - \alpha \quad \frac{\alpha}{2} \quad \alpha$$

$$\begin{aligned} \text{Total no. of moles at equilibrium} &= 1 - \alpha + \frac{\alpha}{2} + \alpha \\ &= \left(1 + \frac{\alpha}{2}\right) \approx 1 \end{aligned}$$

$$\text{Now, } k_p = \frac{(P_A)^{1/2} \cdot (P_B)}{P_{\text{AB}_2}} = \frac{\left(\frac{\alpha}{2} P\right)^{1/2} \times (\alpha \times P)}{\left(\frac{1 - \alpha}{1} \times P\right)}$$

$$\Rightarrow k_p = \frac{\alpha^{3/2} \times P^{1/2}}{\sqrt{2}}$$

$$\Rightarrow \alpha^{3/2} = \frac{\sqrt{2} \cdot k_p}{(P)^{1/2}}$$

$$\Rightarrow \alpha = \frac{(2)^{1/3} \times (k_p)^{2/3}}{(P)^{1/3}} = \left(\frac{2 \times k_p^2}{P}\right)^{1/3}$$

10. Λ_m v/s \sqrt{C} graph is given with negative slope for an electrolyte, Then molar conductance for the same electrolyte at infinite dilution shows-

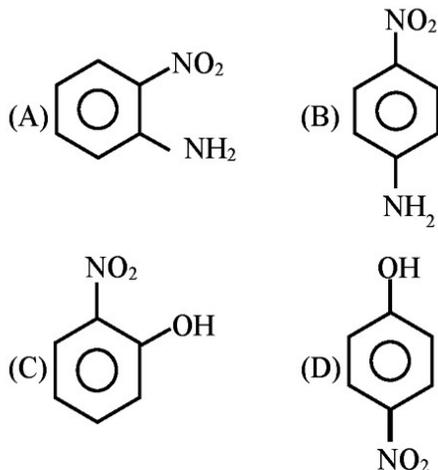
- (1) Small increase (2) Small decrease
 (3) Sharp increase (4) Sharp decrease

Ans. (1)

Sol. λ_m V/S \sqrt{C} graph is linear with negative slope for an electrolyte \Rightarrow strong electrolyte.

Then molar conductance for the same electrolyte at infinite dilution show small increase.

16. Which of the following compounds are steam volatile?



- (1) A & B (2) A & C
 (3) C & D (4) B & D

Ans. (2)

Sol. Due to intramolecular H-Bonding in orthonitrophenol, boiling point decreases and due to intermolecular H-Bonding in paranitrophenol, boiling point increases. They are steam volatile.

17. Match the following column and choose the correct option.

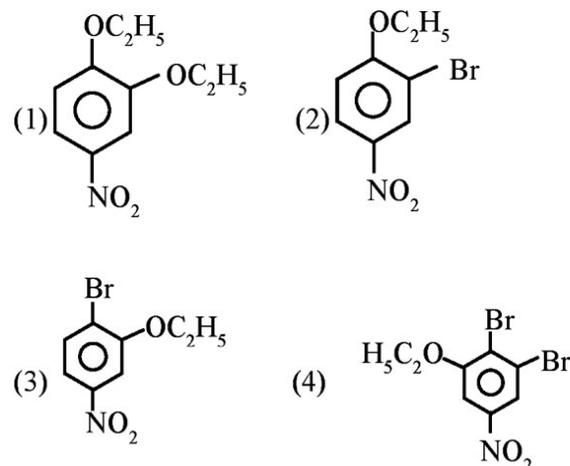
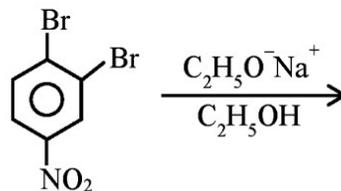
Column-I		Column-II	
(A)	Cellulose	(P)	α -1,6-linkage (animal starch)
(B)	Amylopectin	(Q)	α -1,4-linkage
(C)	Maltose	(R)	β -1,4-linkage
(D)	Glycogen	(S)	α -1,6-linkage (Plant starch)

- (1) A-(S), B-(R), C-(Q), D-(P)
 (2) A-(S), B-(R), C-(P), D-(Q)
 (3) A-(R), B-(S), C-(Q), D-(P)
 (4) A-(R), B-(Q), C-(S), D-(P)

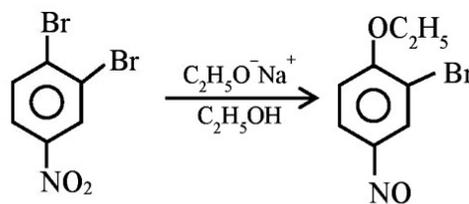
Ans. (3)

Sol. (A) Cellulose : β -1,4-linkage
 (B) Amylopectin : α -1,6-linkage (Plant starch)
 (C) Maltose : α -1,4-linkage
 (D) Glycogen : α -1,6-linkage (animal starch)

18.



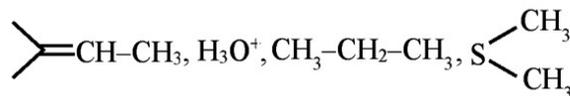
Ans. (2)



Sol.

19. How many of following species can act as nucleophile?

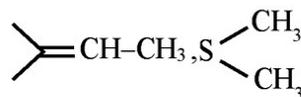
Ph-SH, OH⁻, CH₂=CH₂,



- (1) 5
 (2) 6
 (3) 7
 (4) 4

Ans. (5)

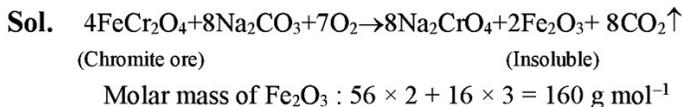
Sol. Nucleophile = Ph-SH, OH⁻, CH₂=CH₂,



SECTION-B

1. Chromite ore + Na_2CO_3 + $\text{O}_2 \rightarrow$ Insoluble product containing Fe. Give the molar mass of insoluble product formed. (Given : Molar mass of Cr = 52 g/mol, Na = 23 g/mol, Fe = 56 g/mol, O = 16 g/mol)

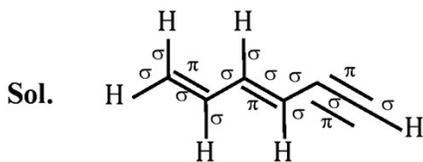
Ans. (160)



2. Calculate the total number of σ and π bond in the given molecule?

Hex-1,3-dien-5-yne

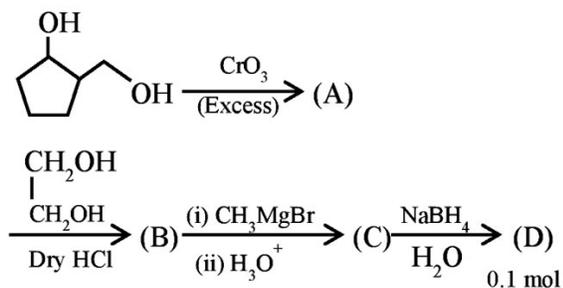
Ans. (15)



Total number of σ -bond = 11

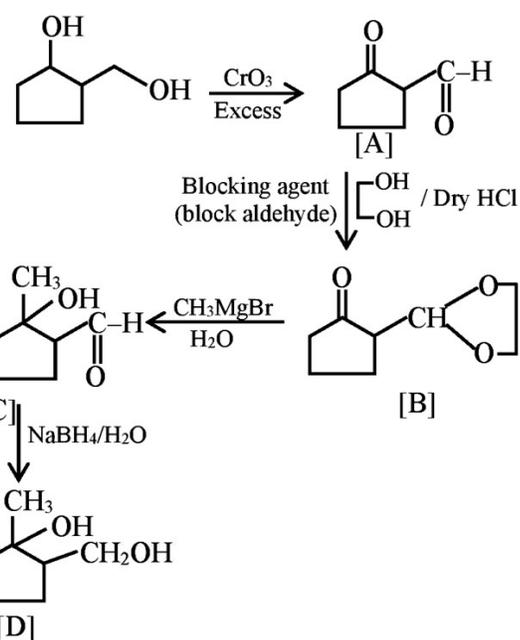
Total number of π -bond = 4

3. Consider the following reaction



Find the mass of final product(D) formed in gm (nearest integer)

Ans. (13)



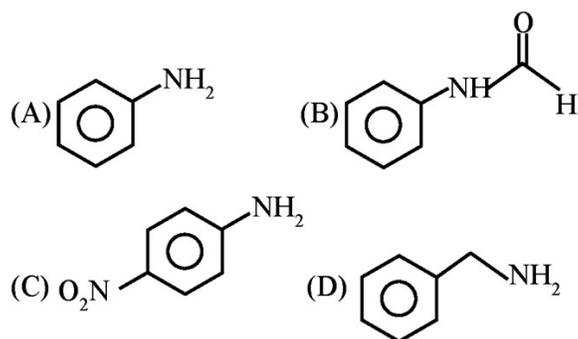
Sol.

Molecular weight of compound (D) = 130 gm/mole

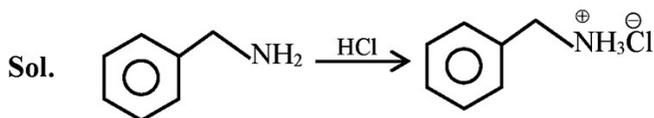
Mole of compound (D) = 0.1

mass of final product (D) = $0.1 \times 130 = 13\text{gm}$

4. One gram of the most basic compound (among the following) reacts with HCl. The mass of HCl consumed in mg is (give your answer as nearest integer):



Ans. (341)



Mass = 1 gm

Mole = $\frac{1}{107}$

Mole of HCl used = $\frac{1}{107}$

Mass of HCl used = $\frac{1}{107} \times 36.5 = 0.341\text{gm}$

= 341mg

SECTION-A

1. The minimum value of n for which the number of integer terms in the binomial expansion of $\left(7^{\frac{1}{3}} + 11^{\frac{1}{12}}\right)^n$ is 183, is

Sol. General term $\rightarrow {}^n C_r \left(7^{\frac{1}{3}}\right)^{n-r} \left(11^{\frac{1}{12}}\right)^r$

$$\Rightarrow {}^n C_r (7)^{\frac{n-r}{3}} (11)^{\frac{r}{12}}$$

For in integral terms, r must be multiple of 12.

i.e. $r = 12\lambda$

given total values of $r = 183$

i.e. maximum value of $r = 12 \times 182 = 2184$

\therefore Minimum value of $n = 2184$

2. $80 \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$

Sol. $80 \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$

$$= 80 \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{9 + 16 \{1 - (1 - \sin 2x)\}} dx$$

$$= 80 \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{25 - 16(\sin x - \cos x)^2} dx$$

Put $\sin x - \cos x = t$

$(\cos x + \sin x) dx = 1 dt$

$$= 80 \int_{-1}^1 \frac{1 dt}{25 - 16t^2}$$

$$= \frac{80 \times 2}{10} \frac{\log \left(\frac{5+4t}{5-4t} \right) \Big|_0^1}{4}$$

$$= 160 \times \frac{1}{2 \times 5} \frac{\ln \left| \frac{5+4t}{5-4t} \right| \Big|_0^1}{4}$$

$$= 4 [\ln|9| - \ln|1|]$$

$$= 4 \ln 9 = 8 \ln 3$$

3. $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} + \hat{k}$. If $\vec{a} \times \vec{c} = \vec{c} \times \vec{b}$ and $(\vec{a} + \vec{c}) \cdot (\vec{b} + \vec{c}) = 168$, then $|\vec{c}|^2$ is equal to

Ans. (308, 77)

Sol. $\vec{a} \times \vec{c} = -\vec{b} \times \vec{c}$

$$\Rightarrow (\vec{a} + \vec{b}) \times \vec{c} = \vec{0}$$

$$\therefore \vec{c} \parallel \vec{a} + \vec{b}$$

$$\vec{c} = \lambda(\vec{a} + \vec{b}) = \lambda(5\hat{i} - 6\hat{j} + 4\hat{k})$$

$$(\vec{a} + \vec{c}) \cdot (\vec{b} + \vec{c}) = 168$$

$$\Rightarrow \vec{a} \cdot \vec{b} + (\vec{a} + \vec{b}) \cdot \vec{c} + |\vec{c}|^2 = 168$$

$$\Rightarrow 6 + 5 + 3 + \lambda(25 + 36 + 16) + \lambda^2 \times 77 = 168$$

$$14 + 77\lambda + 77\lambda^2 = 168$$

$$\Rightarrow \lambda^2 + \lambda - 2 = 0 \Rightarrow \lambda = -2, 1$$

$$\therefore |\vec{c}|^2 = \lambda^2 \times 77$$

For $\lambda = -2$

$$|\vec{c}|^2 = 4 \times 77 = 308$$

For $\lambda = 1$

$$|\vec{c}|^2 = 1 \times 77 = 77$$

4. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^3 + 6k^2 + 11k + 5}{(k+3)!}$ is equal to

- (1) $\frac{5}{3}$ (2) $\frac{8}{3}$ (3) 3 (4) $\frac{7}{3}$

Ans. (1)

Sol. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{(k+1)(k+2)(k+3) - 1}{(k+3)}$

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k} - \frac{1}{k+3} \\ &= \left(\frac{1}{1} - \frac{1}{4} \right) + \left(\frac{1}{2} - \frac{1}{5} \right) + \left(\frac{1}{3} - \frac{1}{6} \right) + \left(\frac{1}{4} - \frac{1}{7} \right) + \dots \\ &= 1 + \frac{1}{2} + \frac{1}{3} \\ &= 1 + \frac{1}{2} + \frac{1}{6} \\ &= \frac{6+3+1}{6} = \frac{10}{6} = \frac{5}{3} \end{aligned}$$

5. $L_1: \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2}, L_2: \frac{x+1}{-1} = \frac{y-2}{-2} = \frac{z}{1}$

Let the line L_3 passes through the point (α, β, γ) perpendicular to L_1 & L_2 and L_3 intersect line L_1 , then $|5\alpha - 11\beta - 8\gamma|$ is equal to

- (1) 25 (2) 18 (3) 16 (4) 20

Ans. (1)

Sol. D. R. S of $L_3 = \vec{m} \times \vec{n}$

$$\begin{aligned} &= -5\hat{i} - 3\hat{j} + \hat{k} \\ &= \langle 5, 3, -1 \rangle \end{aligned}$$

$$\frac{x-\alpha}{5} = \frac{y-\beta}{3} = \frac{z-\gamma}{-1} = t$$

$$L_1 = \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2} = k$$

$$5t + \alpha = k + 1 \Rightarrow \alpha = k + 1 - 5t$$

$$3t + \beta = -k + 2 \Rightarrow \beta = -k + 2 - 3t$$

$$-t + \gamma = 2k + 1 \Rightarrow 2k + 1 + t$$

$$= |5(k+1-5t) - 11(-k+2-3t) - 8(2k+1+t)|$$

$$= |5 - 22 - 8| = 25$$

6. Sum of first three terms of an A.P. with integral common difference is 54 and sum of first twenty terms lies between 1600 to 1800, find a_{11}

- (1) 108 (2) 90 (3) 111 (4) 115

Ans. (2)

Sol. $a + (a + d) + (a + 2d) = 54$

$$3a + 3d = 54$$

$$a + d = 18$$

$$a = 18 - d \quad \dots(1)$$

$$1600 < \frac{20}{2}(2a + 19d) < 1800$$

$$160 < 2a + 19d < 180$$

$$160 < 36 - 2d + 19d < 180$$

$$124 < 17d < 144$$

$$7.29 < d < 8.47$$

$$d = 8$$

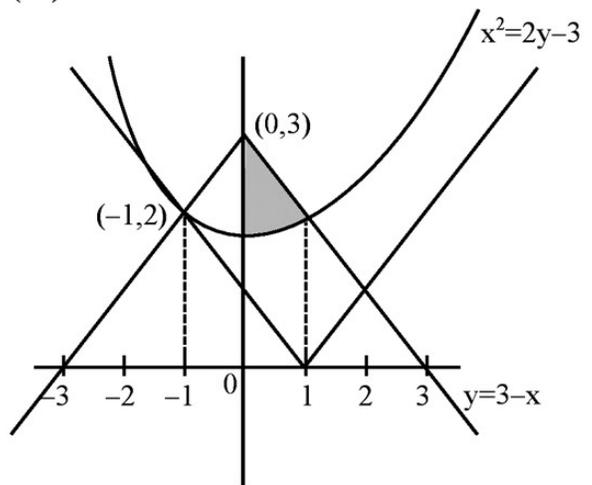
$$\text{from (1) } a = 10$$

$$a_{11} = a + 10d$$

$$= 10 + 80 = 90$$

7. Area enclosed by $y \geq |x - 1|$, $y + |x| \leq 3$, $x^2 \leq 2y - 3$ is A, then $6A$ (in sq. units)

Ans. (10)



Sol.

$$A = 2 \left[\int_0^1 (3-x) dx - \int_0^1 \left(\frac{x^2+3}{2} \right) dx \right]$$

$$A = 2 \left[\left(3x - \frac{x^2}{2} \right)_0^1 - \frac{1}{2} \left[\frac{x^3}{3} + 3x \right]_0^1 \right]$$

$$A = 2 \left[\frac{5}{2} - \frac{1}{2} \left(\frac{10}{3} \right) \right]$$

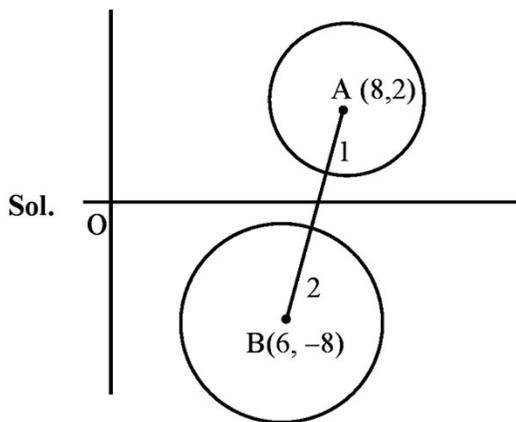
$$A = 5 - \frac{10}{3} = \frac{5}{3}$$

$$6A = 10$$

8. Let $|z_1 - 8 - 2i| \leq 1$ and $|z_2 - 6 + 8i| \leq 2$, and minimum value of $|z_1 - z_2|$ is equal to $\sqrt{a} - b$ where $a, b \in \mathbb{N}$, then the value of $a + b$ is equal to

- (1) 104 (2) 102
(3) 107 (4) 108

Ans. (3)



$$\therefore AB = \sqrt{104}$$

$$\therefore \text{minimum value of } |z_1 - z_2| = \sqrt{104} - 3$$

$$a + b = 104 + 3 = 107$$

9. If R be a relation defined on $\left(0, \frac{\pi}{2}\right)$ such that $xRy \Rightarrow \sec^2 x - \tan^2 y = 1$, then the relation R is

- (1) Equivalence relation
(2) Reflexive and transitive only
(3) Symmetric and transitive only
(4) Neither reflexive nor transitive

Ans. (1)

Sol. (i) xRx

$$\sec^2 x - \tan^2 x = 1 \quad \text{Reflexive}$$

$$(ii) \sec^2 x - \tan^2 y = 1$$

$$\text{Then } \sec^2 y - \tan^2 x = 1 + \tan^2 y - (\sec^2 x - 1)$$

$$= 2 - 1 = 1 \quad \text{Symmetric}$$

$$(iii) \sec^2 x - \tan^2 y = 1$$

$$\sec^2 y - \tan^2 z = 1$$

add

$$\sec^2 x - \tan^2 z = 1 \quad \text{Transitive}$$

Hence, equivalence relation.

10. Number of 7-digit numbers made with the digit 1, 2, 3 such that sum of the digits is 11 is equal to

Ans. (161)

Sol. (i) Number of numbers created using

$$1111133 = \frac{7!}{5!2!} = 21$$

(ii) Number of numbers created using

$$1111223 = \frac{7!}{4!2!} = 105$$

(iii) Number of numbers created using

$$1112222 = \frac{7!}{3!4!} = \frac{7 \cdot 5 \cdot 6}{6} = 35$$

Total = 161

11. If $\cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x - 1)$, then find the sum of all values of x .

- (1) 1 (2) $\frac{1}{2}$ (3) 0 (4) $\frac{3}{2}$

Ans. (3)

$$\text{Sol. } \cos^{-1} x = \pi + \frac{\pi}{2} - \cos^{-1} x + \sin^{-1}(2x - 1)$$

$$2 \cos^{-1} x = \frac{3\pi}{2} + \sin^{-1}(2x - 1)$$

$$\cos(2 \cos^{-1} x) = \cos\left(\frac{3\pi}{2} + \sin^{-1}(2x - 1)\right)$$

$$2x^2 - 1 = \sin(\sin^{-1}(2x - 1))$$

$$2x^2 - 1 = 2x - 1$$

$$x = 0, 1$$

$x = 0$ is only one solution

But $x = 1$, is rejected

12. The minimum value of $p \in \mathbb{N}$ such that

$$\lim_{x \rightarrow 0^+} x \left(\left[\frac{1}{x} \right] + \left[\frac{2}{x} \right] + \dots + \left[\frac{p}{x} \right] \right) - x^2 \left(\left[\frac{1}{x^2} \right] + \left[\frac{2}{x^2} \right] + \dots + \left[\frac{9}{x^2} \right] \right) \geq 1$$

is equal to

Ans. (10)

$$\text{Sol. } \lim_{x \rightarrow 0^+} x \left(\frac{1}{x} + \frac{2}{x} + \dots + \frac{p}{x} \right) + x^2 \left(\frac{1}{x^2} + \frac{2}{x^2} + \dots + \frac{9}{x^2} \right) - x \left(\left[\frac{1}{x} \right] + \left[\frac{2}{x} \right] + \dots + \left[\frac{p}{x} \right] \right) + x^2 \left(\left[\frac{1}{x^2} \right] + \dots + \left[\frac{9}{x^2} \right] \right) \geq 1$$

$$\Rightarrow (1+2+3+\dots+p) - \frac{9 \times (9+1)}{2} \geq 1$$

$$\frac{p(p+1)}{2} \geq 46$$

Where $p \in \mathbb{N}$ & min value = 10

$$13. \text{ If } L = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \sin 4x \\ 1 + \sin^2 x & \cos^2 x & \sin 4x \\ \sin^2 x & \cos^2 x & 1 + \sin 4x \end{vmatrix} \text{ and}$$

$L_{\min} = m$ and $L_{\max} = M$, then $|M^4 - m^4|$ is equal to

$$(1) 79 \quad (2) 78 \quad (3) 80 \quad (4) 76$$

Ans. (3)

$$\text{Sol. } L = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \sin 4x \\ 1 + \sin^2 x & \cos^2 x & \sin 4x \\ \sin^2 x & \cos^2 x & 1 + \sin 4x \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$= \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \sin 4x \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$$

$$= \sin^2 x(-1) - (1 + \cos^2 x)(1 - 0) + \sin 4x(-1)$$

$$= -\sin^2 x(-1) - 1 - \cos^2 x - \sin 4x$$

$$= -1 - 1 - \sin 4x$$

$$= -2 - \sin 4x$$

$$L_{\min} = -2 - 1 = -3 = m$$

$$L_{\max} = -2 + 1 = -1 = M$$

$$|M^4 - m^4| = |(-1)^4 - (-3)^4| = 80$$

14. If α, β real numbers such that $\sec^2(\tan^{-1} \alpha) + \operatorname{cosec}^2(\cot^{-1} \beta) = 36$ and $\alpha + \beta = 8$, then $\alpha^3 + \beta^3$ is equal to

$$(1) 146 \quad (2) 152 \quad (3) 148 \quad (4) 150$$

Ans. (2)

$$\text{Sol. } \tan^{-1} \alpha = A \quad \alpha = \tan A$$

$$\cot^{-1} \beta = B \quad \beta = \cot B$$

$$\sec^2 A + \operatorname{cosec}^2 B = 36$$

$$1 + \tan^2 A + 1 + \cot^2 B = 36$$

$$\alpha^2 + \beta^2 = 34$$

$$\therefore \alpha^2 + \beta^2 + 2\alpha\beta = 64 \Rightarrow \alpha\beta = 15$$

$$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$$

$$= 8(34 - 15)$$

$$= 8(19) = 152$$

15. How many 6 letter words can be formed using the word "MATHS" such that any letter can be used maximum two times?

$$(1) 6400 \quad (2) 8100$$

$$(3) 10000 \quad (4) 9824$$

Ans. (2)

Sol. MM AA TT HH SS

$$2 \text{ Alike, } 4 \text{ Distinct} : {}^5C_1 \times 1 \frac{6!}{2!} = 1800$$

2 Alike, 2 Alike, 2 Distinct :

$${}^5C_2 \times {}^3C_2 \times \frac{6!}{2!2!} = 5400$$

$$2 \text{ Alike, } 2 \text{ Alike, } 2 \text{ Alike} : {}^5C_3 \times \frac{6!}{2!2!2!} = 900$$

$$\text{Total} = 8100$$

16. A triangle is formed by three lines $2x + 3y - 5 = 0$, $x + y - 1 = 0$, $3x + 4y - 7 = 0$. Let (h, k) be the image of the centroid of ΔABC in the line $2x + 4y - 7 = 0$, then $h^2 + k^2 + kh$ is equal to

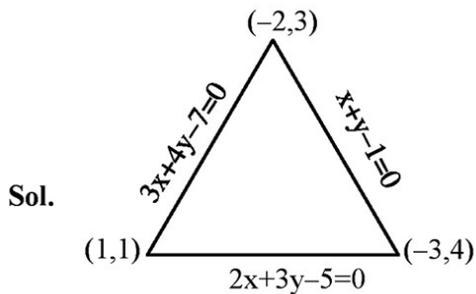
(1) $\frac{903}{225}$

(2) $\frac{223}{225}$

(3) $\frac{100}{23}$

(4) $\frac{10006}{225}$

Ans. (1)



$$\text{centroid} = \left(\frac{1 + (-3) + (-2)}{3}, \frac{1 + 4 + 3}{3} \right)$$

$$= \left(-\frac{4}{3}, \frac{8}{3} \right)$$

Triangle in $2x + 4y - 7 = 0$

$$\frac{k + \frac{4}{3}}{2} = \frac{k - \frac{8}{3}}{4} = -2 \frac{\left(-\frac{5}{3} + \frac{32}{3} - 7 \right)}{4 + 16}$$

$$h = -\frac{23}{15}$$

$$k = \frac{34}{15}$$

$$h^2 + k^2 + hk = \left(-\frac{23}{15} \right)^2 + \left(\frac{34}{15} \right)^2 - \frac{23}{15} \times \frac{34}{15}$$

$$= \frac{903}{225}$$

17. Two parabolas having common focus at $(4, 3)$ intersect at points A and B. Find the value of $(AB)^2$ given that directrices of these parabolas are along X-axis and Y-axis respectively.

Sol. $(x - 4)^2 + (y - 3)^2 = x^2 \quad \dots(1)$

$$(x - 4)^2 + (y - 3)^2 = y^2 \quad \dots(2)$$

$A(x_1, y_1)$ and $B(x_2, y_2)$

Both satisfy the above two equations

Also, from (1) & (2) we get that $x_1^2 = y_1^2$

$$\Rightarrow x_1 = y_1$$

So $(x_1 - 4)^2 + (x_1 - 3)^2 = x_1^2$ and

$$(x_2 - 4)^2 + (x_2 - 3)^2 = x_2^2$$

After solving

$$x_1 + x_2 = 14$$

$$x_1 + x_2 = 25$$

$$\text{So, } (AB)^2 = \left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \right)^2$$

$$= 2(x_1 - x_2)^2$$

$$= 2 \left[\sqrt{(x_1 + x_2)^2 - 4x_1x_2} \right]^2$$

$$= 2 \left[\sqrt{(14)^2 - 4 \times 25} \right]^2$$

$$= 2(196 - 100)$$

$$= 192$$