MOMENTUM

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Answers & Solutions

JEE Advanced

Paper-1

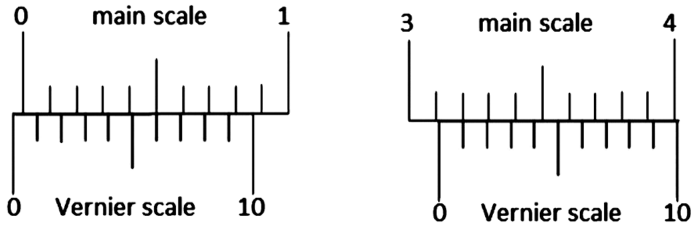
PART-1 : PHYSICS

SECTION 1

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

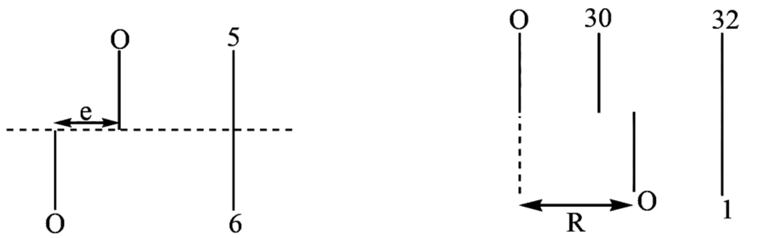
1. The smallest division on the main scale of a Vernier calipers is 0.1 cm. Ten divisions of the Vernier scale correspond to nine divisions of the main scale. The figure below on the left shows the reading of this calipers with no gap between its two jaws. The figure on the right shows the reading with a solid sphere held between the jaws. The correct diameter of the sphere is



(A) 3.07 cm (B) 3.11 cm (C) 3.15 cm (D) 3.17 cm

Key: **C**

Sol



$e + 5(\text{MSD}) = 6(\text{VSD})$ $\qquad R + 1(\text{VSD}) = 32(\text{MSD})$

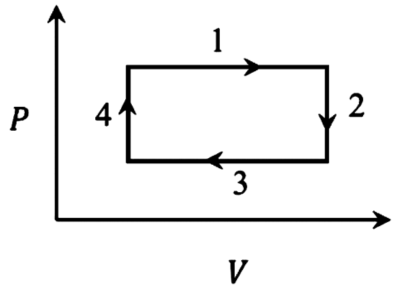
Required diameter $= e + R$

$= (6\text{VSD} - 5\text{MSD}) + (32\text{MSD} - 1\text{VSD})$

$= 27\text{MSD} + 5\text{VSD} \quad = 27\text{mm} + 4.5\text{mm}$

$= 3.15$ cm

2. An ideal gas undergoes a four step cycle as shown in the P – V diagram below. During this cycle, heat is absorbed by the gas in



(A) steps 1 and 2 (B) steps 1 and 3 (C) steps 1 and 4 (D) steps 2 and 4

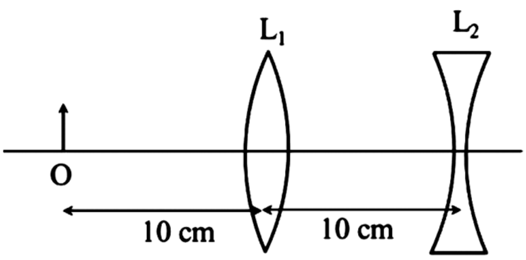
Key: **C**

Sol $\rightarrow$ in the path (1) volume of an ideal gas is increasing at constant pressure.

$\because$ $V \propto T = dQ$ is +ve (absorbed)

In the path (4) pressure of an ideal gas is increasing at constant volume $\because$ $P \propto T = dQ$ is +ve (absorbed)

3. An extended object is placed at point O, 10 cm in front of a convex lens L_1 and a concave lens L_2 is placed 10 cm behind it, as shown in the figure. The radii of curvature of all the curved surfaces in both the lenses are 20 cm. The refractive index of both the lenses is 1.5. The total magnification of this lens system is



(A) 0.4 (B) 0.8 (C) 1.3 (D) 1.6

Key: **B**

Sol For lens L_1

Side note :

$$\frac{1}{f_1}=(u-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=\frac{1}{2}\left(\frac{2}{20}\right)=\frac{1}{20}$$

$$\frac{1}{f_2}=(u-1)\left(-\frac{2}{20}\right)=-\frac{1}{20}$$

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}$$

$$\frac{1}{v_1}+\frac{1}{10}=+\frac{1}{20}$$

$$\frac{1}{v_1}=-\frac{1}{20}=m_1=\frac{v_1}{u_1}=2$$

For lens L_2

$$\frac{1}{v_2}=-\frac{1}{20}-\frac{1}{30}=-\frac{1}{12}$$

$$m_2=\frac{v_2}{u_2}=\frac{-12}{-30}$$

$$m=m_1\times m_2=\frac{24}{30}=\frac{4}{5}=0.8$$

4. A heavy nucleus Q of the half-life 20 minutes undergoes alpha-decay with probability of 60% and beta-decay with probability of 40%. Initially, the number of Q nuclei is 1000. The number of alpha-decays of Q in the first one hour is

(A) 50 (B) 75 (C) 350 (D) 525

Key: **D**

Sol at t = 0 $N_0=1000$; α-decay and β-decay are in 3 : 1 ratio.

At t = 20 min $\rightarrow$ $\frac{\alpha-\text{decay}}{300}$ $\frac{\beta-\text{decay}}{200}$

At t = 40 min $\rightarrow$ 300 + 150 200 + 100

At t = 60 min $\rightarrow$ $\frac{300+150+75}{525}$ $\frac{200+100+50}{350}$

SECTION 2

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;
Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 5 and 6

Question Stem

A projectile is thrown from a point O on the ground at an angle $45°$ from the vertical and with a speed $5\sqrt{2}\ m/s$. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x meters from the point O. The acceleration due to gravity $g = 10\ m/s^2$.

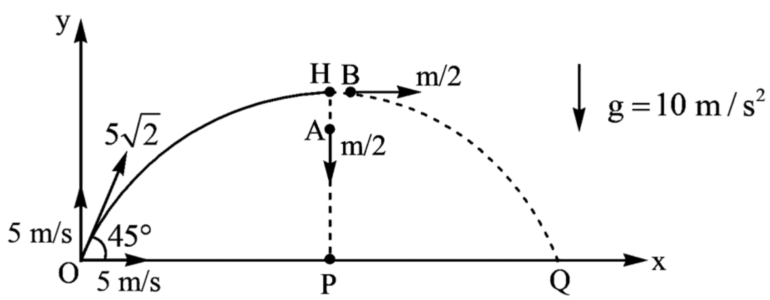
5. The value of t is _______ .

Key: **0.5**

6. The value of x is _______ .

Key: **7.5**

5,6. Sol



$\rightarrow$ from O to H time of ascent $= \frac{U_y}{g} = \frac{5}{10} = 0.5$ sec

$\rightarrow$ at highest point $mv_x = \frac{m}{2} v_x^1$

$v_x^1 = 10\ m/s$

$\rightarrow$ one part A of mass m/2 is falling vertically down in 0.5 sec

$\Rightarrow$ part B is not having vertical component of velocity, it is falling freely in gravity

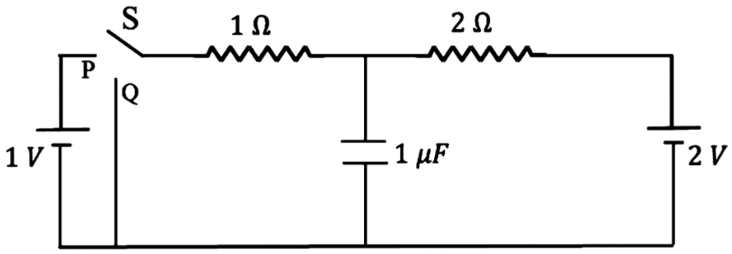
$\Rightarrow$ $t = 0.5$ sec

$OQ = OP + PQ = 5\times 0.5 + 10\times 0.5 = 7.5m$

Question Stem for Question Nos. 7 and 8

Question Stem

In the circuit shown below, the switch S is connected to position P for a long time so that the charge on the capacitor becomes q_1 μC. Then S is switched to position Q. After a long time, the charge on the capacitor is q_2 μC.



7. The magnitude of q_1 is _______ .

Key: **1.33**

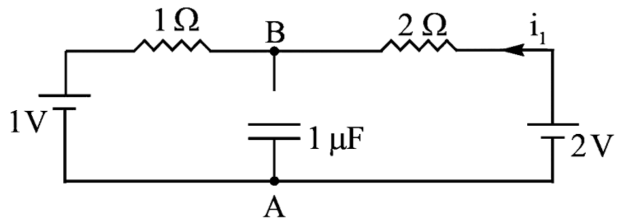
8. The magnitude of q_2 is _______ .

Key: **0.66 to 0.67**

7,8. Sol

Before shifting the switch S from P to Q.

$t \rightarrow \infty$



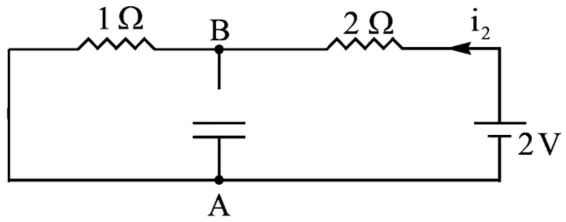
$i_1 = \frac{1}{3}$ Amp $\Rightarrow V_A + 2 - 2i_1 = V_B \Rightarrow V_{AB} = 2 - \frac{2}{3} = \frac{4}{3}$ V

$q_1 = V_{AB} \cdot C$

$q_1 = \frac{4}{3} \mu C$

$\rightarrow$ after shifting the switch S from P to Q.

$t \rightarrow \infty$



$i_2 = \frac{2}{3}$ Amp

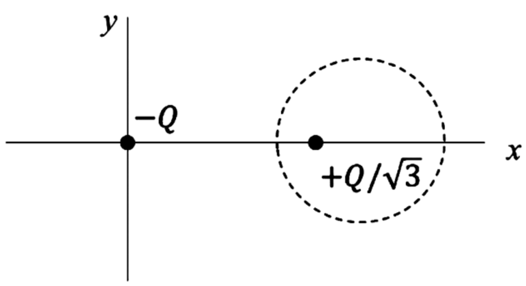
$V_A + 2 - 2 \times \frac{2}{3} = V_B$

$V_{AB} = \frac{2}{3} V$ $q_2 = V_{AB} \cdot C = \frac{2}{3} \mu C$

Question Stem for Question Nos. 9 and 10

Question Stem

Two point charges $-Q$ and $+Q/\sqrt{3}$ are placed in the xy-plane at the origin (0, 0) and a point (2, 0), respectively, as shown in the figure. This results in an equipotential circle of radius R and potential V = 0 in the xy-plane with its center at (b, 0). All lengths are measured in meters.



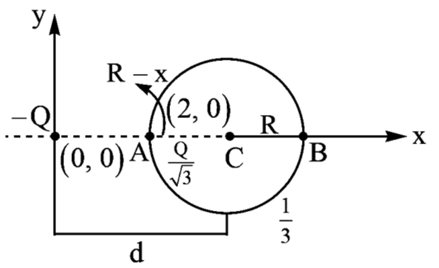
9. The value of R is _______ meter.

Key: **1.73**

10. The value of b is ______ meter.

Key: **3**

9,10. Sol Let 'P' be the point (2, 0) and CP = x



$V_A = V_B = 0$ $V_A = \frac{-kQ}{(d-R)} + \frac{kQ}{\sqrt{3}(R-x)} = 0$

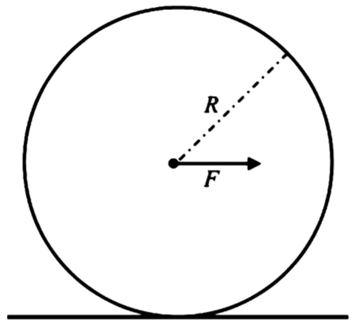
$V_B = \frac{-kQ}{(d+R)} + \frac{kQ}{\sqrt{3}(R+x)} = 0$ $\Rightarrow d = \sqrt{3}R,\ x = d - 2 = \frac{R}{\sqrt{3}}$ $\Rightarrow R\sqrt{3},\ d = 3$

SECTION 3

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get −2 marks.

11. A horizontal force F is applied at the centre of mass of a cylindrical object of mass m and radius R, perpendicular to its axis as shown in the figure. The coefficient of friction between the object and the ground is μ. The centre of mass of the object has an acceleration a. The acceleration due to gravity is g. Given that the object rolls without slipping, which of the following statement(s) is(are) correct ?



(A) For the same F, the value of a does not depends on whether the cylinder is solid or hollow

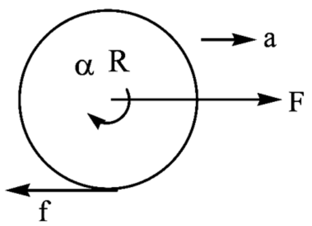
(B) For a solid cylinder, the maximum possible value of a is $2\mu g$

(C) The magnitude of the frictional force on the object due to the ground is always μmg

(D) For a thin-walled hollow cylinder, $a = \frac{F}{2m}$

Key: **BD**

Sol



$F - f = ma$

$f \cdot R = I\alpha$

$a = R\alpha$

$FR = mR^2\alpha + I\alpha \quad \Rightarrow \alpha = \frac{FR}{I + mR^2}$

$a = \frac{FR^2}{I + mR^2}$

$f = \frac{IF}{I + mR^2} = \frac{Ia}{R^2}$

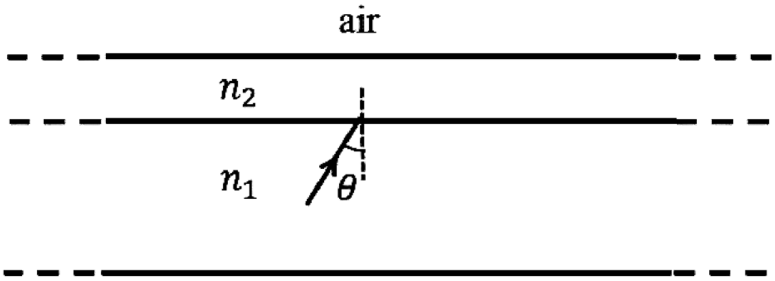
(B) for solid cylinder $I = \frac{mR^2}{2}$

$a = \dfrac{FR^2}{\frac{3}{2}mR^2} = \dfrac{2F}{3m} \quad \Rightarrow \quad a = \dfrac{fR^2}{I} = \dfrac{\mu mgR^2}{\frac{mR^2}{2}} = 2\mu g$

(C) $f = \mu N = \mu mg$ is the limiting friction and not always the value.

(D) for hollow cylinder, $a = \dfrac{FR^2}{2mR^2}$; $I = mR^2 = \dfrac{F}{2m}$

12. A wide slab consisting of two media of refractive indices n_1 and n_2 is placed in air as shown in the figure. A ray of light is incident from medium n_1 to n_2 at an angle θ, where $\sin\theta$ is slightly larger than $1/n_1$. Take refractive index of air as 1. Which of the following statement(s) is(are) correct ?



(A) The light ray enters air if $n_2 = n_1$

(B) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 < n_1$

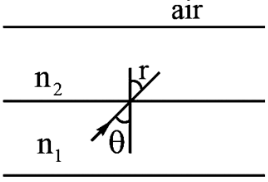
(C) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 > n_1$

(D) The light ray is reflected back into the medium of refractive index n_1 if $n_2 = 1$

Key: **BCD**

Sol $n_1 \sin\theta = n_2 \sin r$

$\sin\theta$ is given slightly greater than $\dfrac{1}{n_1}$. If 'n_2' medium was not present then ray would have suffered TIR at n_1 and air interface.



$\Rightarrow \quad \sin\theta = \dfrac{n_2}{n_1}\sin r \quad \dfrac{n_2}{n_1}\sin r \geq \dfrac{1}{n_1}$

$\sin r \geq \frac{1}{n_2} \Rightarrow$ As $\sin r$ is slightly greater than $\frac{1}{n_2}$, the ray will come back into n_2 and finally n_1 medium irrespective of value of n_2. So both B, C options are correct.

Also if $n_1 = n_2$ the light ray does not enter into air

13. A particle of mass M = 0.2 kg is initially at rest in the xy-plane at a point $(x = -\ell, y = -h)$, where $\ell = 10$ m and $h = 1$ m. The particle is accelerated at time t = 0 with a constant acceleration $a = 10\ m/s^2$ along the positive x-direction. Its angular momentum and torque with respect to the origin, in SI units, are represented by $\vec{L}$ and $\vec{\tau}$, respectively. $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the positive x, y and z-directions, respectively. If $\hat{k} = \hat{i} \times \hat{j}$ then which of the following statement(s) is(are) correct ?

(A) The particle arrives at the point $(x = \ell, y = -h)$ at time t = 2 s

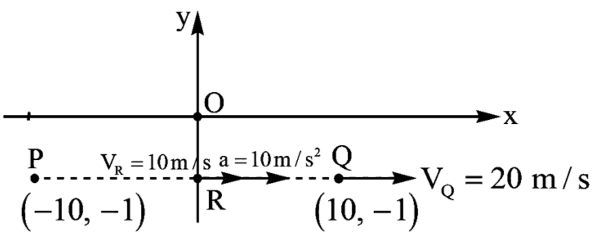
(B) $\vec{\tau} = 2\hat{k}$ when the particle passes through the point $(x = \ell, y = -h)$

(C) $\vec{L} = 4\hat{k}$ when the particle passes through the point $(x = \ell, y = -h)$

(D) $\vec{\tau} = \hat{k}$ when the particle passes through the point $(x = 0, y = -h)$

Key: **ABC**

Sol



$PQ = \frac{1}{2}at^2 = \frac{1}{2} \times 10 \times t^2 = 20$

$t = 2$ sec, $\quad \vec{\tau} = -\hat{j} \times ma\,\hat{i} = ma\,\hat{k} = 2\hat{k}$

$\vec{L} = -\hat{j} \times 0.2 \times 20\,\hat{j} = 4\hat{k}$

14. Which of the following statement(s) is(are) correct about the spectrum of hydrogen atom ?

(A) The ratio of the longest wavelength to the shortest wavelength in Balmer series is 9/5

(B) There is an overlap between the wavelength ranges of Balmer and Paschen series

(C) The wavelengths of Lyman series are given by $\left(1 + \frac{1}{m^2}\right)\lambda_0$, where λ_0 is the shortest wavelength of Lyman series and m is an integer

(D) The wavelength ranges of Lyman and Balmer series do not overlap

Key: **AD**

Sol (A) $\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$

$\frac{1}{\lambda_2}=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{5R}{36}$ (1)

$\frac{1}{\lambda_3}=R\left(\frac{1}{2^2}\right)=\frac{R}{4}$ (2)

$\frac{(2)}{(1)}=\frac{\lambda_2}{\lambda_3}=\frac{\frac{R}{4}}{\frac{5R}{36}}=\frac{9}{5}$ (A) is correct

(B) Balmer series belongs to visible
Pasehen series belongs to I.R region.

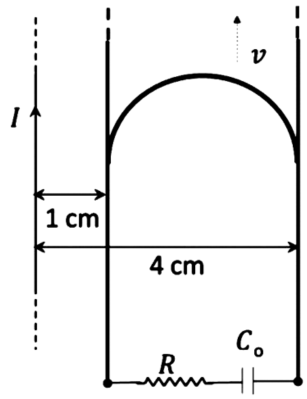
(C) $\frac{1}{\lambda}=R\left(1-\frac{1}{m^2}\right)$; $\frac{1}{\lambda_0}=R$ when $m\to\infty \Rightarrow \lambda=\frac{1}{R}\frac{1}{\left(1-\frac{1}{m^2}\right)}=\frac{\lambda_0}{\left(1-\frac{1}{m^2}\right)}$

(D) Lyman series belongs to U.V
Balmer series belongs to visible.

15. A long straight wire carries a current, I = 2 ampere. A semi-circular conducting rod is placed beside it on two consuding parallel rails of negligible resistance. Both the rails are parallel to the wire. The wire, the rod and the rails lie in the same horizontal plane, as shown in the figure. Two ends of the semi-circular rod are at distances 1 cm and 4 cm from the wire. At time t = 0, the rod starts moving on the rails with a speed v = 3.0 m/s (see the figure).

A resistor $R = 1.4\ \Omega$ and a capacitor $C_0 = 5.0\ \mu F$ are connected in series between the rails. At time t = 0, C_0 is uncharged. Which of the following statement(s) is(are) correct ?

$\left[\mu_0 = 4\pi\times10^{-7}\text{ SI units. Take }\ln 2 = 0.7\right]$



(A) Maximum current through R is 1.2×10^{-6} ampere

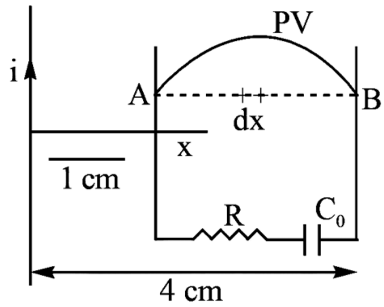
(B) Maximum current through R is 3.8×10^{-6} ampere

(C) Maximum charge on capacitor C_0 is 8.4×10^{-12} coulomb

(D) Maximum charge on capacitor C_0 is 2.4×10^{-12} coulomb

Key: **AC**

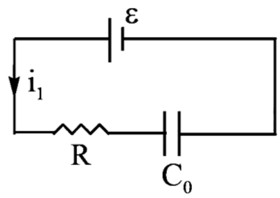
Sol



→ Here V is instantaneous velocity

Emf $AB=\int\left(\overline{V}\times\overline{B}\right)\cdot dx$ $\qquad = V\int\frac{\mu_0 i}{2\pi x}dx$

$\varepsilon=\frac{\mu_0 iV}{2\pi}\ell n(4)$



$\varepsilon-i_1R=\frac{q}{C}\Rightarrow i_1=\frac{dq}{dt}$ (1)

$\varepsilon=\frac{\mu_0 i}{2\pi}(\ell n4)(V)$

For maximum charge $\frac{dq}{dt}=i_1=0$

From (1) $\varepsilon=\frac{q}{C}$

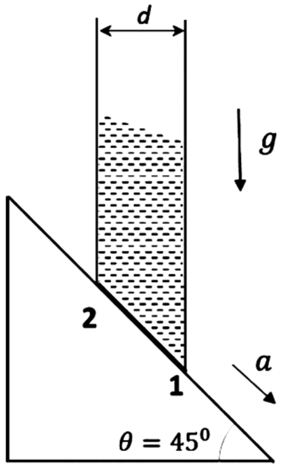
$I_{max}=\frac{\mu_0 iV}{2\pi}\ell n(4)\times C_0$

$=8.4\times10^{-12}\,C$

For maximum current q = 0

$\Rightarrow\quad i_1=\frac{\varepsilon}{R}\ ;\ \frac{\mu_0 iV}{2\pi}\frac{\ell n(4)}{R}=1.2\times10^{-6}$ Amp

16. A cylindrical tube, with its base as shown in the figure, is filled with water. It is moving down with a constant acceleration a along a fixed inclined plane with angle $\theta = 45°$. P_1 and P_2 are pressure at points 1 and 2, respectively, located at the base of the tube Let $\beta = (P_1 - P_2)/(\rho g d)$, where ρ is density of water, d is the inner diameter of the tube and g is the acceleration due to gravity. Which of the following statement(s) is(are) correct ?



(A) $\beta = 0$ when $a = g/\sqrt{2}$

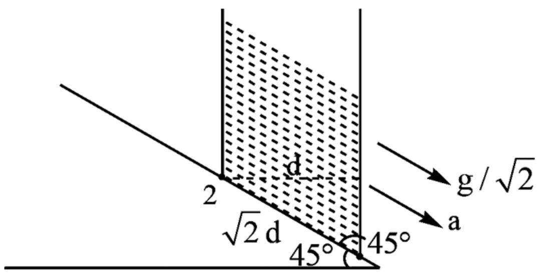
(B) $\beta > 0$ when $a = g/\sqrt{2}$

(C) $\beta = \frac{\sqrt{2}-1}{\sqrt{2}}$ when $a = g/2$

(D) $\beta = \frac{1}{\sqrt{2}}$ when $a = g/2$

Key: **AC**

Sol



$$P_1 + \rho(\sqrt{2}\,d)a - \rho\frac{g}{\sqrt{2}}(\sqrt{2}\,d) = P_2$$

$$P_1 - P_2 = \rho d(-\sqrt{2}\,a + g)$$

$$\beta = \frac{P_1 - P_2}{\rho g d} = \left(-\frac{\sqrt{2}\,a}{g} + 1\right)$$

if $\beta = 0 \Rightarrow a = \frac{g}{\sqrt{2}}$

if $a = \frac{g}{2}$ then $\beta = \frac{\sqrt{2}-1}{\sqrt{2}}$

SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct integer is entered;
  Zero Marks : 0 In all other cases.

17. An α-particle (mass 4 amu) and a singly charged sulfur ion (mass 32 amu) are initially at rest. They are accelerated through a potential V and then allowed to pass into a region of uniform magnetic field which is normal to the velocities of the particles. Within this region, the α-particle and the sulphur ion move in circular orbits of radii r_α and r_S, respectively. The ratio (r_S / r_α) is _______ .

Key: **4**

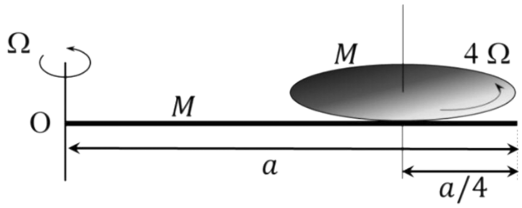
Sol $q_\alpha = +2e \quad ; \quad q_s = -e$

$$\frac{p^2}{2m} = qV \Rightarrow P \propto \sqrt{qm}$$

$$r \propto \frac{P}{q} \propto \sqrt{\frac{m}{q}} \qquad \frac{r_\alpha}{r_s} = \sqrt{\frac{m_\alpha}{m_s} \times \frac{q_s}{q_\alpha}} \qquad \frac{r_s}{r_\alpha} = 4$$

18. A thin rod of mass M and length a is free to rotate in horizontal plane about a fixed vertical axis passing through point O. A thin circular disc of mass M and of radius $a/4$ is pivoted on this rod with its center at a distance $a/4$ from the free end so that it can rotate freely about its vertical axis, as shown in the figure. Assume that both the rod and the disc have uniform density and they remain horizontal during the motion. An outside stationary observer finds the rod rotating with an angular velocity Ω and the disc rotating about its vertical axis with angular velocity 4Ω. The total angular momentum of the system about the point O is $\left(\frac{Ma^2\Omega}{48}\right)n$.

The value of n is _______ .



Key: **49**

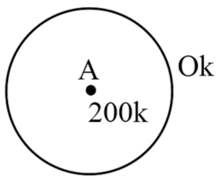
Sol $I_0 = \frac{ma^2}{3}\Omega + \frac{m}{2}\left(\frac{a}{4}\right)^2 4\Omega + m\left(\frac{3a}{4}\right)^2 \Omega$

$= ma^2\Omega\left[\frac{1}{3}+\frac{1}{8}+\frac{9}{16}\right] = ma^2\Omega\left[\frac{16+6+3\times9}{48}\right] = \frac{49}{48}\, ma^2\,\Omega \qquad \Rightarrow n = 49$

19. A small object is placed at the centre of a large evacuated hollow spherical container. Assume that the container is maintained at 0 K. At time t = 0, the temperature of the object is 200 K. The temperature of the object becomes 100 K at $t = t_1$ and 50 K at $t = t_2$. Assume the object and the container to be ideal black bodies. The heat capacity of the object does not depend on temperature. The ratio (t_2/t_1) is _____ .

Key: **9**

Sol



$\sigma Ae\left(T^4 - O^4\right) = ms\frac{dT}{dt}$

$\rightarrow \int\frac{dT}{T^4} \propto \int dt$

$T_1 = 100k \qquad \frac{1}{T_0^3} - \frac{1}{T_1^3} = k\, t_1$

$T_2 = 50k \qquad \frac{1}{T_0^3} - \frac{1}{T_2^3} = k\, t_2$

$\frac{t_2}{t_1} = 9$

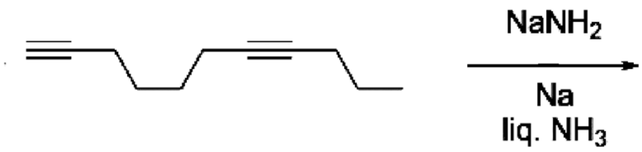
PART-2 : CHEMISTRY

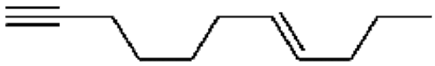
SECTION 1

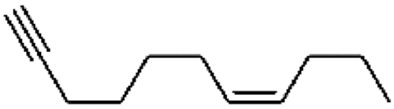
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

1. The major product formed in the following reaction is



(A) 
(B) 

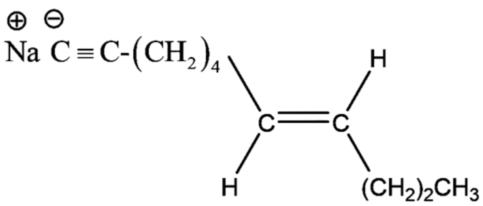
(C) 
(D) 

Key. B

Sol:

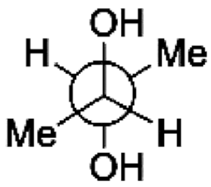
$$CH \equiv C-(CH_2)_4-C \equiv C-(CH_2)_2-CH_3 \xrightarrow{NaNH_2} \overset{\oplus}{Na}\overset{\ominus}{C} \equiv C-(CH_2)_4-C \equiv C-(CH_2)_2CH_3$$

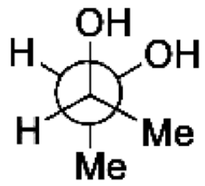
$\downarrow$ Na in liq. NH_3 (Birch Reduction)

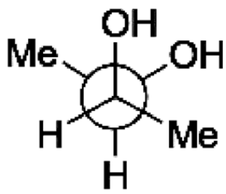


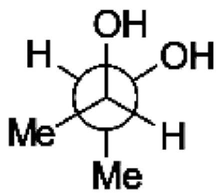
Birch reduces alkynes to trans alkenes trans except terminal alkyne ∴ Answer is B.

2. Among the following, the conformation that corresponds to the most stable conformation of *meso*-butane-2,3-diol is

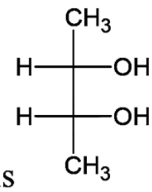
(A) - 

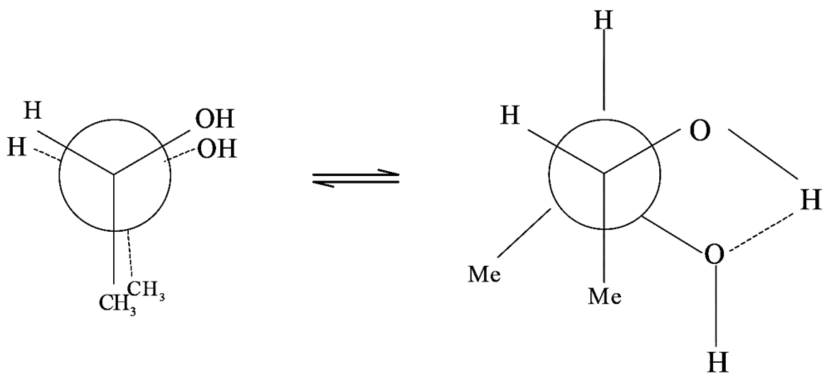
(B) - 

(C) - 

(D) - 

Key B

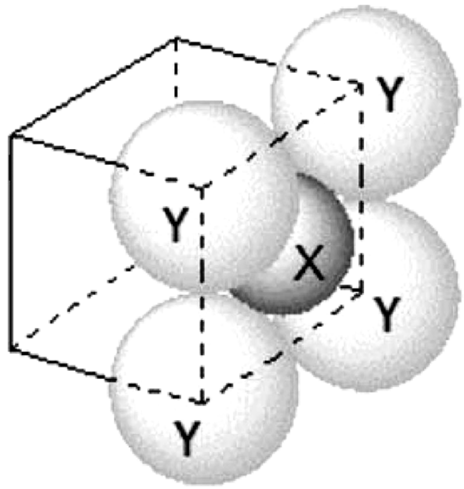
Sol: Meso – Butane-2, 3-diol is 



gauche

3. For the given close packed structure of a salt made of cation X and anion Y shown below (ions of only one face are shown for clarity), the packing fraction is approximately

(packing fraction $= \frac{\text{packing efficiency}}{100}$)



(A) 0.74 (B) 0.63 (C) 0.52 (D) 0.48.

Key B

Sol: Number of Y atoms per unit cell = 1

Number of X atoms per unit cell = 3

$2r_y = a$ $\qquad \frac{r_y}{a} = \frac{1}{2}$

$2r_x + 2r_y = \sqrt{2}a$ $\qquad r_x = \frac{\sqrt{2}a - a}{2} = \left(\frac{\sqrt{2}-1}{2}\right)a$

$$\text{P.F.} = \frac{\frac{4}{3}\pi r^3{}_y + 3\times\frac{4}{3}\pi r_x^3}{a^3} \qquad \text{P.F.} = \frac{4}{3}\pi\left(\frac{1}{8} + 3\left(\frac{\sqrt{2}-1}{2}\right)^3\right)$$

$$\text{P.F} = \frac{4}{3}\pi\left(\frac{1+0.21}{8}\right) = \frac{\pi(1.21)}{6} \qquad \text{P.F.} = 0.63$$

4. The calculated spin only magnetic moments of $\left[Cr(NH_3)_6\right]^{3+}$ and $\left[CuF_6\right]^{3-}$ in BM, respectively, are

(Atomic numbers of Cr and Cu are 24 and 29, respectively)

(A) 3.87 and 2.84 (B) 4.90 and 1.73

(C) 3.87 and 1.73 (D) 4.90 and 2.84

Key A

Sol: $\left[Cr(NH_3)_6\right]^{3+}$, $Cr^{3+} = t_{2g}^3\ eg^o$ $\qquad \mu = \sqrt{3(3+2)} = 3.87$

$\left[CuF_6\right]^{3-}$; $\qquad Cu^{3+} = t_{2g}^6\ eg^2$ $\qquad \mu = \sqrt{2(2+2)} = 2.84$

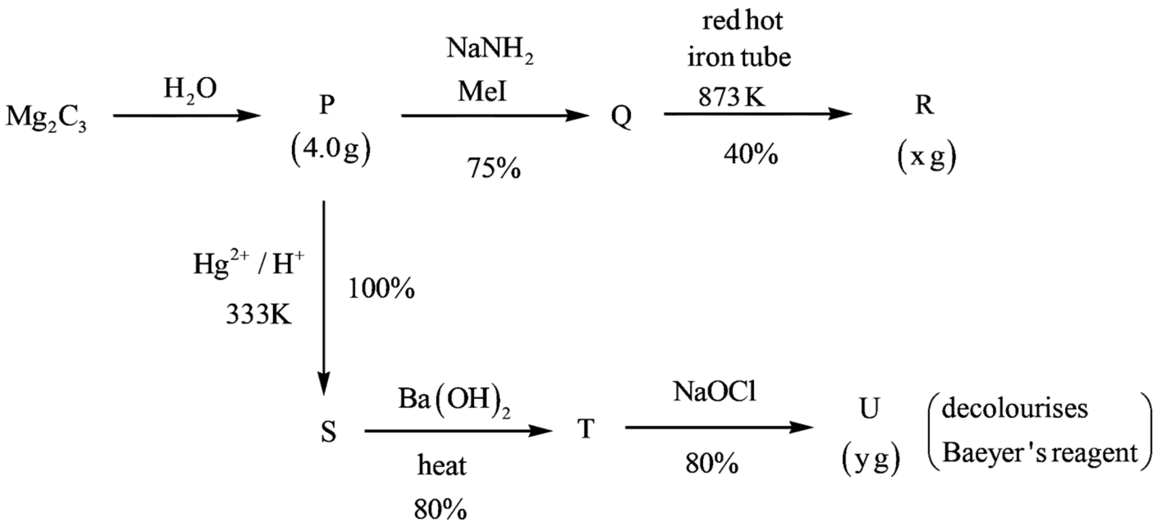
SECTION 2

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;
Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 5 and 6
Question Stem

For the following reaction scheme, percentage yields are given along the arrow:

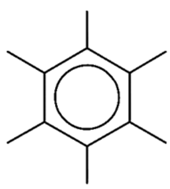


X g and y g are mass of R and U, respectively.

(Use: Molar mass (in g mol^{-1}) of H, C and O as 1, 12 and 16, respectively)

5. The value of x is ______.

Key **1.62** gm

$3CH \equiv C - CH_3 \longrightarrow 3CH_3 - C \equiv C - CH_3 \longrightarrow$ 

Sol:

$\frac{4}{40}=0.1$ $\frac{1}{3}$

0.1 0.75×0.1 $\left(0.4\times 0.75\times\frac{1}{3}\times 162\,\text{gm}\right)$

= 1.62 gm

6. The value of y is ______.

Key **3.2**

Sol: Molar mass of u = 100 gm

For 100 % yield

From 80 gm of P $\longrightarrow$ 100 gm of u is obtained

From 4 gm of P $\longrightarrow$ $\frac{100}{80}\times 4\,\text{gm}$ of u will be obtained

= 5 gm of u will be obtained

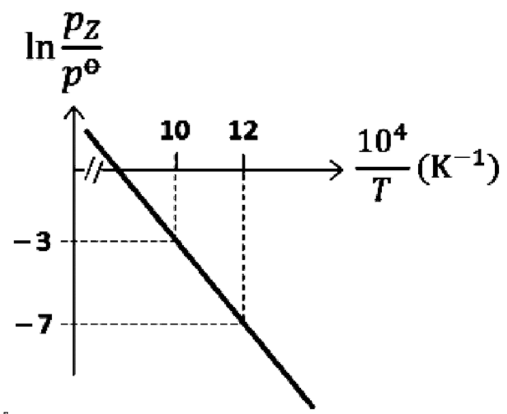
But as the yield is $\frac{100}{100}\times\frac{80}{100}\times\frac{80}{100}$

u obtained is = $5\times\frac{100}{100}\times\frac{80}{100}\times\frac{80}{100}$ $=\frac{5\times 8\times 2}{5\times 5}$ =3.2 gm

Question stem for Question Nos. 7 and 8

Question Stem

For the reaction, $X(s)\rightleftharpoons Y(s)+Z(g)$, the plot of $\ln\frac{p_z}{p^{\ominus}}$ versus $\frac{10^4}{T}$ is given below (in solid line), where p_z is the pressure (in bar) of the gas Z at temperature T and $p^{\ominus}=1\,\text{bar}$.



(Given, $\frac{d(\ln K)}{d\left(\frac{1}{T}\right)} = -\frac{\Delta H^{\ominus}}{R}$, where the equilibrium constant, $K = \frac{p_z}{p^{\ominus}}$ and the gas constant, $R = 8.314\,J\,K^{-1}\,mol^{-1}$)

7. The value of standard enthalpy, $\Delta H^{\ominus}$ (in $kJ\,mol^{-1}$) for the given reaction is_____.

Key **166.28**

Sol: $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$$-RT\ell n\left(\frac{P_z}{P_0}\right) = \Delta H^\circ - T\Delta S^\circ$$

$$\ell n\left(\frac{P_z}{P^\circ}\right) = -\frac{\Delta H^\circ}{RT} + \frac{\Delta S^\circ}{R}$$

$$\text{Slope} = -\frac{\Delta H^\circ}{R} = -10^4 \times \left(\frac{4}{2}\right)$$

$$\Delta H^\circ = 20 \times 10^3 \times R\,J$$

$$\Delta H^\circ = 20 \times 8.314\,KJ$$

$$\Delta H^\circ = 166.28\,KJ/mol$$

8. The value of $\Delta S^{\ominus}$ (in $kJ\,mol^{-1}$) for the given reaction, at 1000 K is_____.

Key : **141.34**

Sol: $\ell n\left(\frac{P_z}{P^\circ}\right) = \frac{-\Delta H^\circ}{RT} + \frac{\Delta S^\circ}{R}$

At $T = 1000\,K \Rightarrow \frac{10^4}{T} = 10$

$$\Rightarrow \ell n\left(\frac{P_z}{P^\circ}\right) = -3$$

$$-3 = \frac{-\Delta H^\circ}{RT} + \frac{\Delta S^\circ}{R}$$

$$-3 = \frac{-20 \times 8.314 \times 10^3}{8.314 \times 1000} + \frac{\Delta S^\circ}{R}$$

$$\Delta S^\circ = 17 \times 8.314 = 141.338$$

$\Delta S^\circ = 141.33\,J/K/mol$ Truncated value

$\Delta S^\circ = 141.34\,J/K/mol$ round off value

Question Stem for Question Nos. 9 and 10

Question Stem

The boiling point of water in a 0.1 molal silver nitrate solution (solution A) is $x\,°C$. To this solution A, an equal volume of 0.1 molal aqueous barium chloride solution is added to make a new solution B. The difference in the boiling points of water in the two solutions A and B is $y \times 10^{-2}\,°C$.

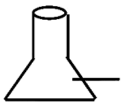
(Assume: densities of the solutions A and B are the same as that of water and the soluble salts dissociate completely.

Use: Molal elevation constant (Ebullioscopic Constant), $K_b = 0.5\,K\,kg\,mol^{-1}$; Boiling point of pure water as $100°C$.)

9. The value of x is_____.

Key **100.1**

Sol:



A

$m = 0.1$ molal $AgNO_3$

$1\,\ell t$ solution

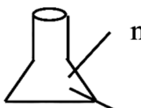
$(\Delta T_b)_A = iK_b m = 2 \times 0.5 \times 0.1 = 0.1$

$X = 100.1°C$

10. The value of $|y|$ is_____.

Key **2.5**

Sol:



B

$m = \frac{0.1}{2}$ molal $AgNO_3$

$m = \frac{0.1}{2}$ molal $BaCl_2$

$(\Delta T_b)_B = K_b[i_1 m_1 + i_2 m_2]$ $\quad = \frac{1}{2}\left[2 \times \frac{0.1}{2} + 3 \times \frac{0.1}{2}\right]$ $\quad = \frac{1}{2}[0.1 + 0.15]$

$(\Delta T_b)_B = \frac{0.25}{2} = 0.125$

$(\Delta T_b)_B - (\Delta T_b)_A = 0.125 - 0.1$

$= 0.025 = 2.5 \times 10^{-2}$

$y = 2.5$

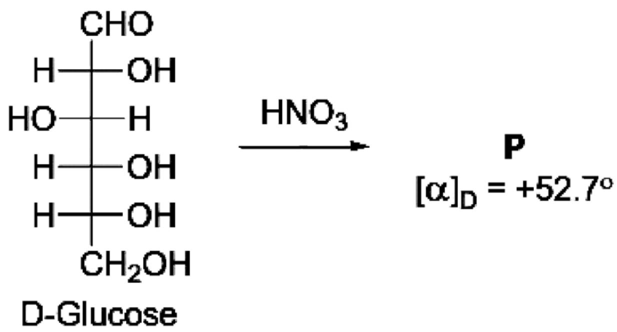
SECTION 3

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

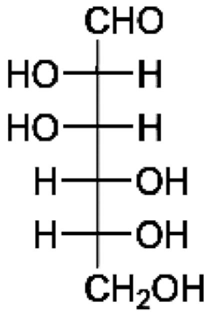
Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If unanswered;
Negative Marks : −2 In all other cases.

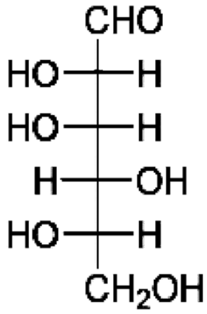
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
choosing any other option(s) will get −2 marks.

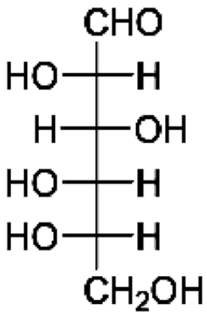
11. Given:

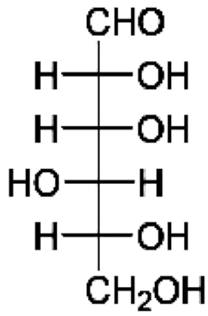


The compound(s), which on reaction with HNO_3 will give the product having degree of rotation, $[\alpha]_D = -52.7°$ is(are)

(A) - 

(B) - 

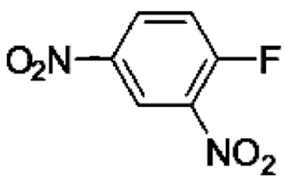
(C) 

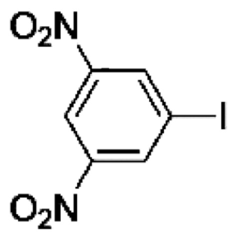
(D) 

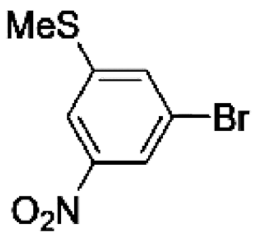
Key CD

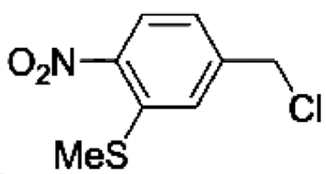
Sol: D-Glucose on oxidation with HNO_3 forms. D-Glucaric or saccharic acid. As the question wants to produce L-Glucaric acid we have start with L-Glucose which is non superimposable mirror image of D-Glucose. i.e., C & D also form the required product.

12. The reaction of Q with PhSNa yields an organic compound (major product) that gives positive Carius test on treatment with Na_2O_2 followed by addition of $BaCl_2$. The correct option(s) for Q is(are)

(A) - 

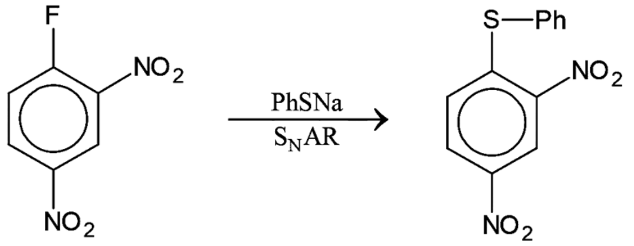
(B) - 

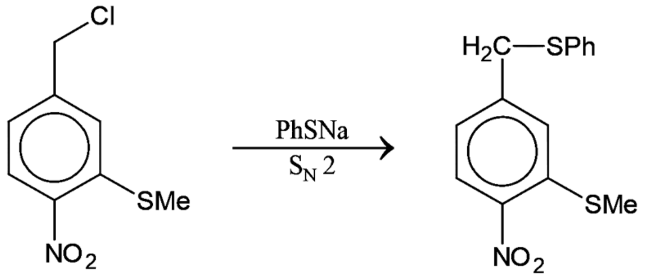
(C) - 

(D) - 

Key AD

Sol:





13. The correct statement(s) related to colloids is(are)

(A) The process of precipitating colloidal sol by an electrolyte is called peptization.

(B) colloidal solution freezes at higher temperature than the true solution at the same concentration.

(C) Surfactants form micelle above critical micelle concentration (CMC).CMC depends on temperature.

(D) Micelles are macromolecular colloids.

Key BC

Sol: (A) Incorrect

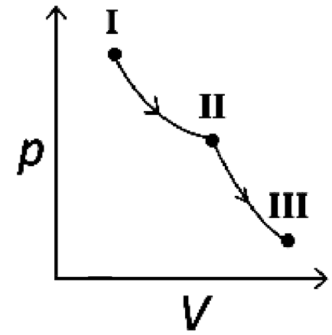
(B) correct

(C) correct

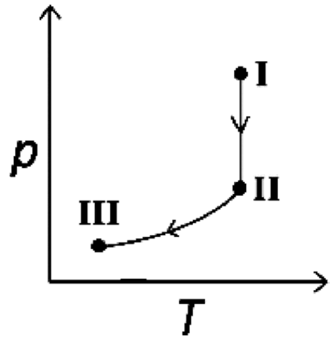
(D) Incorrect

14. An ideal gas undergoes a reversible isothermal expansion from state I to state II followed by a reversible adiabatic expansion from state II to state III. The correct plot(s) representing the changes from state I to state III is(are)

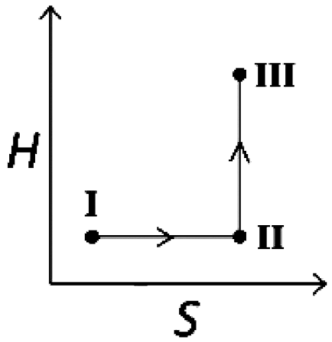
(p: pressure, V: volume, T: temperature, H : enthalpy, S: entropy)



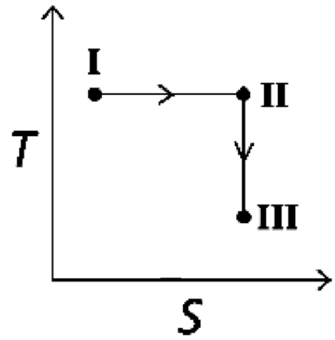
(A)



(B)

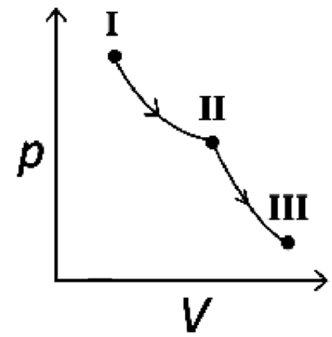


(C)

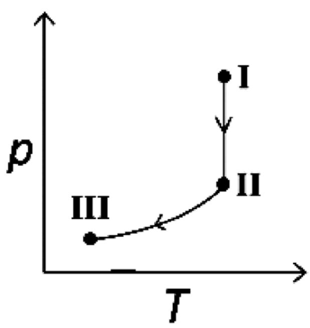


(D)

Key ABD



Sol: (A) - Correct



(B)

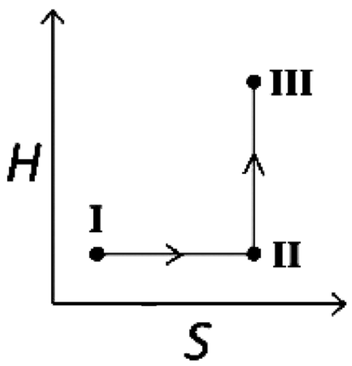
For II to III

$P^{1-r}T^{r} = \text{constant}$

$P\,T^{\frac{r}{1-r}} = \text{constant}$

$\frac{dP}{dr} = \text{constant}\left(\frac{r}{r-1}\right)T^{\frac{1}{r-1}} = +ve$

correct



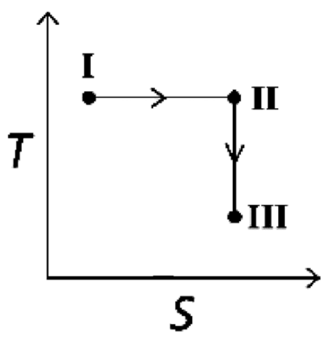
(C) -

Since enthalpy change of ideal gas only depends upon temperature $I \longrightarrow II$ (correct)

But In adiabatic expansion temperature will decrease so enthalpy should decrease

$II \longrightarrow III$ (incorrect)

So the graph is incorrect



(D) -

$I \longrightarrow II$ correct

$II \longrightarrow III$ correct

So the graph is correct

15. The correct statement(s) related to the metal extraction processes is(are)

(A) A mixture of PbS and PbO undergoes self-reduction to produce Pb and SO_2.

(B) In the extraction process of copper from copper pyrites, silica is added to produce copper silicate.

(C) Partial oxidation of sulphide ore of copper by roasting, followed by self-reduction produces blister copper.

(D) In cyanide process, zinc powder is utilized to precipitate gold from $Na[Au(CN)_2]$.

Key ACD

Sol: $\Rightarrow PbS + PbO \longrightarrow Pb + SO_2$

$\Rightarrow Zn + Na[Au(CN)_2] \longrightarrow Au + Na_2[Zn(CN)_4]$

16. A mixture of two salts is used to prepare a solution S, which gives the following results:

$$\text{White precipitate(s) only} \xleftarrow[\text{Room temperature}]{\text{Dilute NaOH(aq)}} \underset{\text{(aq solution of the salts)}}{S} \xrightarrow[\text{Room temperature}]{\text{Dilute HCl (aq)}} \text{White precipitate(s) only}$$

The correct option(s) for the salt mixture is(are)

(A) $Pb(NO_3)_2$ and $Zn(NO_3)_2$

(B) $Pb(NO_3)_2$ and $Bi(NO_3)_3$

(C) $AgNO_3$ and $Bi(NO_3)_3$

(D) $Pb(NO_3)_2$ and $Hg(NO_3)_2$

Key AB

Sol:

$Pb^{2+} + Cl^- \longrightarrow \underset{\text{white}}{PbCl_2} \downarrow$

$Ag^+ + OH^- \longrightarrow \underset{\text{Brown}}{Ag_2O} \downarrow$

$Bi^{3+} + OH^- \longrightarrow \underset{\text{white}}{Bi(OH)_3} \downarrow$

$Hg^{2+} + OH^- \longrightarrow \underset{\text{Brownish Red.}}{HgO} \downarrow$

SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

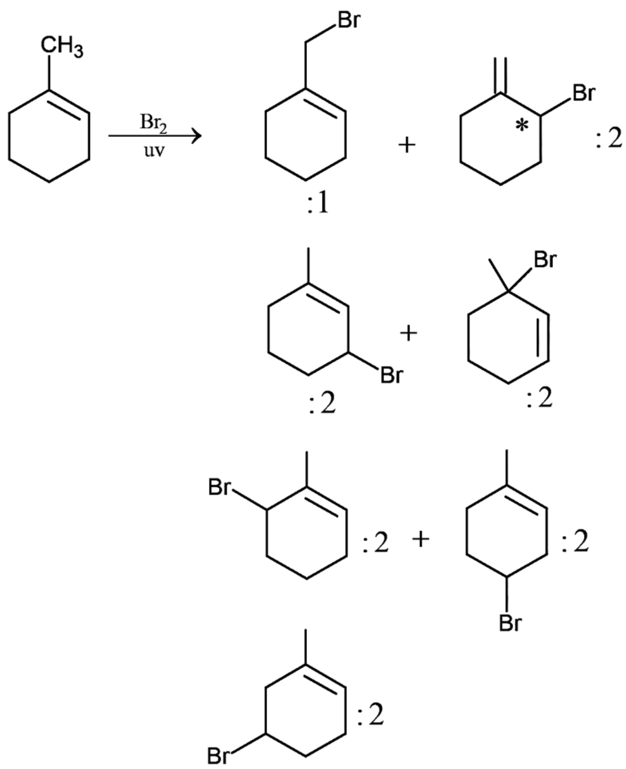
Full Marks : +4 If ONLY the correct integer is entered;

Zero Marks : 0 In all other cases.

17. The maximum number of possible isomers (including stereosiomers) which may be formed on *mono*-bromination of 1-methylcyclohex-1-ene using Br_2 and UV light is _____.

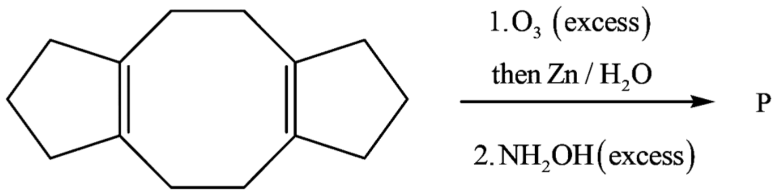
Key 13

Sol:



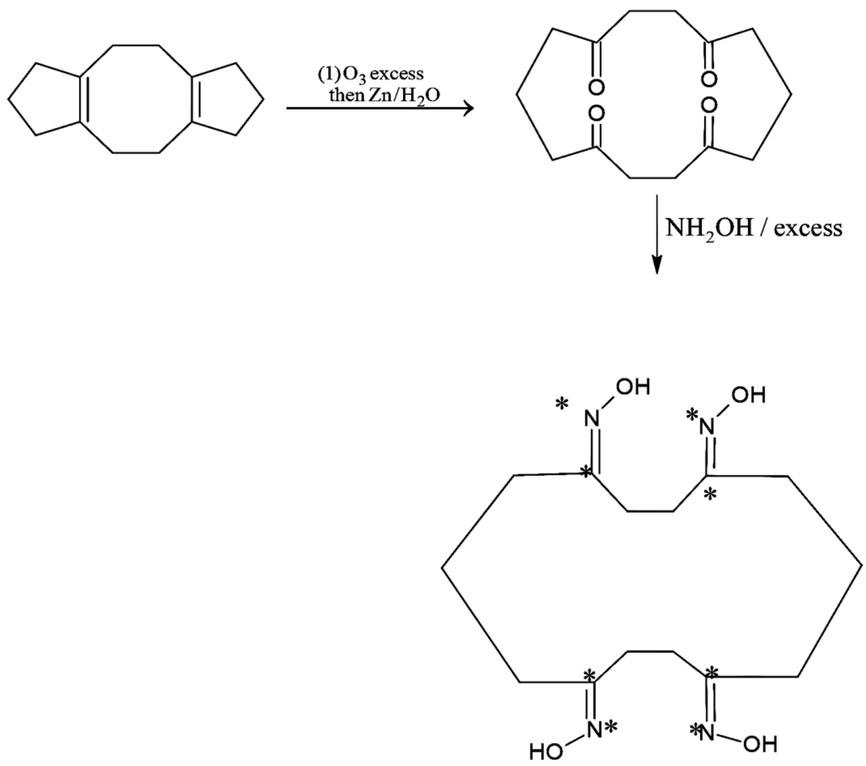
Total isomers obtained = 13

18. In the reaction given below, the total number of atoms having sp^2 hybridization in the major product P is____.



Key 8

Sol:



Total 8 atoms $4N + 4C$ have sp^2 hybridization

As NCERT gives oxygen of phenol to be sp^3 so similarly here too it should be sp^3.

19. The total number of possible isomers for $[Pt(NH_3)_4Cl_2]Br_2$ is____.

Key 6

Sol: Considering structural and Sterio

(1) $[Pt(NH_3)_4Br_2]Cl_2 \longrightarrow$ cis & trans

(2) $[Pt(NH_3)_4ClBr]ClBr \longrightarrow$ cis & trans

(3) $[Pt(NH_3)_4Cl_2]Br_2 \longrightarrow$ cis & trans

SECTION 1

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

1. Consider a triangle Δ whose two sides lie on the x-axis and the line $x+y+1=0$. If the orthocenter of Δ is (1, 1), then the equation of the circle passing through the vertices of the triangle Δ is

(A) $x^2+y^2-3x+y=0$ (B) $x^2+y^2+x+3y=0$

(C) $x^2+y^2+2y-1=0$ (D) $x^2+y^2+x+y=0$

Key: B

SOL:

$H(1,1)=O.C$

Property : Image of O.C w.r.t any side lie on circum-circle of the triangle

Given sides X-axis i.e. $y=0$ & $x+y+1=0$.

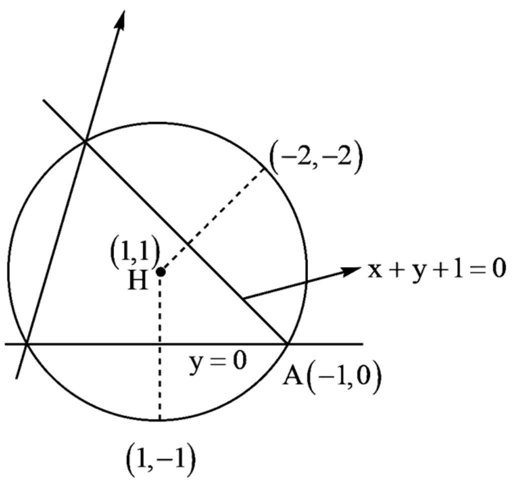


Image of $H(1,1)$ w.r.t $y=0$ is $P(1,-1)$

Image of $H(1,1)$ w.r.t $x+y+1=0$ be $Q(\alpha,\beta)$

PART-3 : MATHEMATICS

SECTION 1

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

1. Consider a triangle Δ whose two sides lie on the x-axis and the line $x+y+1=0$. If the orthocenter of Δ is (1, 1), then the equation of the circle passing through the vertices of the triangle Δ is

(A) $x^2+y^2-3x+y=0$ (B) $x^2+y^2+x+3y=0$

(C) $x^2+y^2+2y-1=0$ (D) $x^2+y^2+x+y=0$

Key: B

SOL:

$H(1,1)=O.C$

Property : Image of O.C w.r.t any side lie on circum-circle of the triangle

Given sides X-axis i.e. y = 0 & $x+y+1=0$.

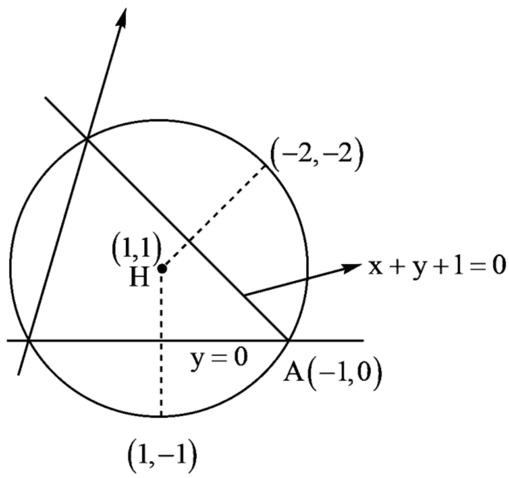


Image of $H(1,1)$ w.r.t $y=0$ is $P(1,-1)$

Image of $H(1,1)$ w.r.t $x+y+1=0$ be $Q(\alpha,\beta)$

$\frac{\alpha-1}{1}=\frac{\beta-1}{1}=\frac{-2(1+1+1)}{1^2+1^2}=-3$, $\alpha=-2,\beta=-2 \Rightarrow Q(-2,-2)$

Point of Intersection of sides y = 0 & $x+y+1=0$ is $A(-1,0)$.

Now $P(1,-1)$, $Q(-2,-2)$ & $A(-1,0)$ lie on circumcircle slope of $\overleftrightarrow{PA}=\frac{1}{-2}$, slope of

$\overleftrightarrow{QA}=\frac{2}{1}$. Product of slopes = -1. $\therefore \overleftrightarrow{PA} \perp \overleftrightarrow{QA}$

$\therefore$ $\overleftrightarrow{PQ}$ is diameter of circumcircle .

circumcircle is $(x-1)(x+2)+(y+1)(y+2)=0$

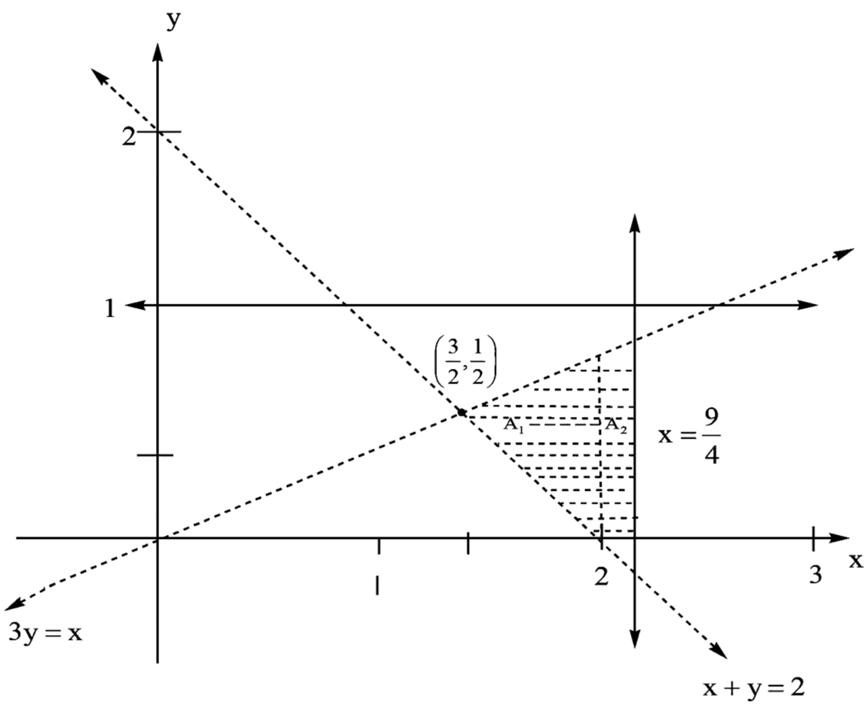
$x^2+x-2+y^2+3y+2=0$, $x^2+y^2+x+3y=0$

2. The area of the region $\left\{(x,y):0\le x\le \frac{9}{4},\ 0\le y\le 1,\ x\ge 3y,\ x+y\ge 2\right\}$ is

(A) $\frac{11}{32}$ (B) $\frac{35}{96}$ (C) $\frac{37}{96}$ (D) $\frac{13}{32}$.

Key: A

Sol:



$$R.A = A_1 + A_2 = \int_{3/2}^{2}\left[\frac{x}{3}-(2-x)\right]dx + \int_{2}^{9/4}\frac{x}{3}dx \qquad = \int_{3/2}^{2}\left(\frac{4x}{3}-2\right)dx + \frac{1}{3}\int_{2}^{9/4} xdx$$

$$\left(\frac{4x^2}{6}-2x\right)_{3/2}^{2} + \frac{1}{3}\left[\frac{x^2}{2}\right]_{2}^{9/4} \qquad = \left[\frac{2}{3}(4)-4\right]-\left[\frac{2}{3}\times\frac{9}{4}-3\right]+\frac{1}{3}\left[\frac{81}{32}-2\right]$$

$$= \left[\frac{8-12}{3}\right]-\left[\frac{9-18}{6}\right]+\frac{1}{3}\left[\frac{81-64}{32}\right] \qquad = \frac{-4}{3}+\frac{3}{2}+\frac{1}{3}\times\frac{17}{32} = \frac{-8+9}{6}+\frac{17}{3\times 32}$$

$$= \frac{16+17}{3\times 32} = \frac{33}{3\times 32} = \frac{11}{32}.$$

3. Consider three sets $E_1 = \{1,2,3\}$, $F_1 = \{1,3,4\}$ and $G_1 = \{2,3,4,5\}$. Two elements are chosen at random, without replacement, from the set E_1, and let S_1 denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements.

Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1,2\}$. Then the value of p is

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{5}$

Key: A

$$\text{Reqiured probability}=\frac{\frac{1}{3}\times\frac{1}{2}\times\frac{1}{10}}{\frac{1}{3}\times\frac{1}{2}\times\frac{1}{10}+\frac{1}{3}\times\left[\frac{1}{2}\times 1\times\frac{1}{10}+\frac{{}^3C_2}{{}^4C_2}\times\frac{1}{6}\right]+\frac{1}{3}\times\left[\frac{2}{3}\times\frac{1}{10}\right]}$$

$$= \frac{\frac{1}{20}}{\frac{1}{20}+\frac{1}{20}+\frac{1}{12}+\frac{1}{15}} = \frac{1}{20}\times\frac{60}{(6+5+4)} = \frac{1}{5}$$

4. Let $\theta_1, \theta_2, ..., \theta_{10}$ be positive valued angles (in radian) such that $\theta_1 + \theta_2 + ... + \theta_{10} = 2\pi$.

Define the complex numbers $z_1 = e^{i\theta_1}$, $z_k = z_{k-1}e^{i\theta_k}$ for k = 2, 3, …, 10, where $i = \sqrt{-1}$.

Consider the statements P and Q given below:

$P: |z_2 - z_1| + |z_3 - z_2| + ... + |z_{10} - z_9| + |z_1 - z_{10}| \le 2\pi$

$Q: |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + ... + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \le 4\pi$

Then,

(A) P is TRUE and Q is FALSE

(B) Q is TRUE and P is FALSE

(C) both P and Q are TRUE

(D) both P and Q are FALSE

Key: C

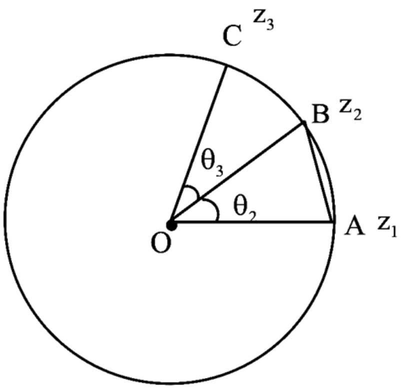
Given $Z_1 = e^{i\theta_1}$

$Z_k = Z_{k-1}e^{i\theta_k}$

For k = 2,3,4…10 $\Rightarrow \frac{Z_2}{Z_1} = e^{i\theta_2}, \frac{Z_3}{Z_2} = e^{i\theta_3}$ ……. $\frac{Z_{10}}{Z_9} = e^{i\theta_{10}}$

Clearly

Length of $\text{Arc}(AB) >$ chord AB



$\Rightarrow |Z_2 - Z_1| \le \left|\text{Arg}\left(\frac{Z_2}{Z_1}\right)\right|$ $\Rightarrow |Z_3 - Z_2| \le \left|\text{Arg}\left(\frac{Z_3}{Z_2}\right)\right|$…………

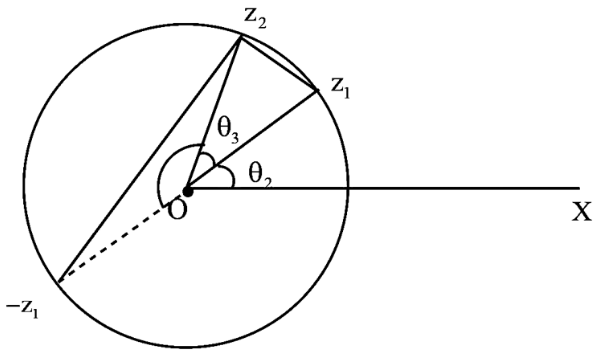
$|Z_1 - Z_{10}| \le \left|\text{Arg}\left(\frac{Z_{10}}{Z_1}\right)\right|$

$P = |Z_2 - Z_1| + |Z_3 - Z_2| + ... + |Z_1 - Z_{10}|$

$P \le \left|\text{Arg}\left(\frac{Z_2}{Z_1}\right)\right| + \left|\text{Arg}\left(\frac{Z_3}{Z_2}\right)\right| + ... + \left|\text{Arg}\left(\frac{Z_{10}}{Z_1}\right)\right|$

$P \le \theta_2 + \theta_3 + ... + \theta_{10} + 2\pi - (\theta_2 + \theta_3 + ... + \theta_{10})$

$\therefore |Z_2 - Z_1| + |Z_3 - Z_2| + ... + |Z_1 - Z_{10}| \le 2\pi$



$\therefore |Z_2 - Z_1| \le \left|\text{Arg}\left(\frac{Z_2}{Z_1}\right)\right|$ and $|Z_2 - (-Z_1)| = |Z_2 + Z_1| \le \left|\text{Arg}\left(\frac{Z_2}{Z_1}\right)\right|$

$\therefore |Z_2^2 - Z_1^2| \le \left|\text{Arg}\left(\frac{Z_2^2}{Z_1^2}\right)\right| = 2\left|\text{Arg}\left(\frac{Z_2}{Z_1}\right)\right|$ $\qquad |z_2^2 - z_1^2| \le 2\left|\text{Arg}\left(\frac{Z_2}{Z_1}\right)\right|$

Similarly $|Z_3^2 - Z_2^2| \le 2\left|\text{Arg}\left(\frac{Z_3}{Z_2}\right)\right|$

$Q = |Z_2^2 - Z_1^2| + |Z_3^2 - Z_2^2| + ...|Z_1^2 - Z_{10}^2| \le 4\pi$

SECTION 2

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;
  Zero Marks : 0 In all other cases.

Question stem for Question Nos. 5 and 6

Three numbers are chosen at random, one after another with replacement, from the set S = {1, 2, 3, …., 100}. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40.

5. The value of $\frac{625}{4}p_1$ is ____

Key: 76.25

$P_1 \rightarrow$ Probability that maximum of chosen number is at least 81

$\Rightarrow P_1 = 1-$ Probability of chosen numbers ≤ 80

$= 1 - \frac{80}{100} \times \frac{80}{100} \times \frac{80}{100}$ ($\because$ with replacement)

$\Rightarrow P_1 = 1 - \frac{64}{125} = \frac{61}{125} \Rightarrow 125P_1 = 61 \Rightarrow \frac{625}{4}P_1 = \frac{305}{4} = 76.25$

6. The value of $\frac{125}{4}p_2$ is ____.

Key: 24.5

$P_2 \rightarrow$ Probability of minimum of chosen number is atmost 40.

$\Rightarrow P_2 = 1-$ Probability of chosen numbers ≥ 41

$= 1 - \frac{60}{100} \times \frac{60}{100} \times \frac{60}{100}$ ($\because$ with replacement)

$P_2 = 1 - \frac{27}{125} = \frac{98}{125} \Rightarrow 125P_2 = 98 \Rightarrow \frac{125}{4}P_2 = \frac{98}{4} = 24.50$

Question stem for Question Nos. 7 and 8

Question Stem-2

Let α, β and γ be real numbers such that the system of linear equations

$x + 2y + 3z = \alpha$

$4x + 5y + 6z = \beta$

$7x + 8y + 9z = \gamma - 1$

is consistent. Let $|M|$ represent the determinant of the matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the square of the distance of the point $(0,1,0)$ from the plane P.

7. The value of $|M|$ is ____.

Key: 1

8. The value of D is ____.

Key: 1.5

7 & 8 Sol: $x + 2y + 3z = \alpha$

$4x + 5y + 6z = \beta$

$7x + 8y + 9z = \gamma - 1$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0$$

$$\Delta_1 = \begin{vmatrix} \alpha & 2 & 3 \\ \beta & 5 & 6 \\ \gamma - 1 & 8 & 9 \end{vmatrix} = 3\begin{vmatrix} \alpha & 2 & 1 \\ \beta & 5 & 2 \\ \gamma - 1 & 8 & 3 \end{vmatrix}$$

$= 3[\alpha(15-16) - 2(3\beta - 2\gamma + 2) + 1(8\beta - 5\gamma + 5)]$

$= 3[-\alpha - 6\beta + 4\gamma - 4 + 8\beta - 5\gamma + 5]$

$= 3[-\alpha + 2\beta - \gamma + 1] = -3[\alpha - 2\beta + \gamma - 1]$

$$\Delta_2 = \begin{vmatrix} 1 & \alpha & 3 \\ 4 & \beta & 6 \\ 7 & \gamma - 1 & 9 \end{vmatrix} = 3\begin{vmatrix} 1 & \alpha & 1 \\ 4 & \beta & 2 \\ 7 & \gamma - 1 & 3 \end{vmatrix}$$

$= 3[(3\beta - 2\gamma + 2) - \alpha(12-14) + 1(4\gamma - 4 - 7\beta)]$

$= 3[3\beta - 2\gamma + 2 + 2\alpha + 4\gamma - 4 - 7\beta]$

$= 3[2\alpha - 4\beta + 2\gamma - 2] = 6[a - 2\beta + \gamma - 1]$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & \alpha \\ 4 & 5 & \beta \\ 7 & 8 & \gamma-1 \end{vmatrix} \qquad = (5\gamma-5-8\beta)-2(4\gamma-4-7\beta)+\alpha(32-35)$$

$= 5\gamma - 5 - 8\beta - 8\gamma + 8 + 14\beta - 3\alpha$

$= -3\alpha + 6\beta - 3\gamma + 3$

For consistent, $\Delta_1 = \Delta_2 = \Delta_3 = 0$

$\Rightarrow \alpha - 2\beta + \gamma - 1 = 0$ ....(1)

(7) $\qquad |M| = \begin{vmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$

$= \alpha(1-0) - 2(\beta - 0) + \gamma(0+1)$

$= \alpha - 2\beta + \gamma$

$= 1$ $\qquad$ (from (1))

$|M| = 1$

(8) $\qquad$ P is plane containing all point (α, β, γ)

For the above system.

$\Rightarrow$ Plane 'P' is $\qquad x - 2y + z - 1 = 0$ ....(2)

Perpendicular distance of $(0,1,0)$ from (2) = $\left| \dfrac{0-2(1)+0-1}{\sqrt{1^2+(-2)^2+1^2}} \right| = \dfrac{3}{\sqrt{6}}$

D = Square of the distance

$= \left(\dfrac{3}{\sqrt{6}}\right)^2 = \dfrac{9}{6} = \dfrac{3}{2} = 1.5$

Question stem for Question Nos. 9 and 10

Question Stem

Consider the lines L_1 and L_2 defined by

$L_1 : x\sqrt{2} + y - 1 = 0$ and $L_2 : x\sqrt{2} - y + 1 = 0$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2. The line $y = 2x + 1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S'. Let D be the square of the distance between R' and S'.

9. The value of λ^2 is ______.

Key: 9

Sol: Let $P(h,k)$

$L_1 = x\sqrt{2} + y - 1 = 0$, $L_2 = x\sqrt{2} - y + 1 = 0$

$$\left|\frac{\left(h\sqrt{2}+k-1\right)}{\sqrt{3}} \times \frac{\left(h\sqrt{2}-k+1\right)}{\sqrt{3}}\right| = \lambda^2$$

$$\left(h\sqrt{2}\right)^2 - (k-1)^2 = \pm 3\lambda^2$$

Locus of $P(h,k)$ is

$$C: 2x^2 - (y-1)^2 = \pm 3\lambda^2 \quad \text{.......(1)}$$

$y = 2x + 1$ cuts curve (1) at R & S $\quad \Rightarrow 2x^2 - (2x)^2 = \pm 3\lambda^2$

$$\Rightarrow 2x^2 = 3\lambda^2 \quad \text{...... (2)}$$

$$x_1 + x_2 = 0,\ x_1x_2 = \frac{-3\lambda^2}{2}$$

$$\& \ 2\left(\frac{y-1}{2}\right)^2 - (y-1)^2 = \pm 3\lambda^2.$$

$$(y-1)^2 = 6\lambda^2 \Rightarrow y^2 - 2y + 1 - 6\lambda^2 = 0 \quad \text{.....(3)}$$

$$y_1 + y_2 = 2,\ y_1y_2 = 1 - 6\lambda^2$$

Given $RS = \sqrt{270}$

$$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{270} \text{ where } R(x_1, y_1) \ \& \ S(x_2, y_2)$$

$$\Rightarrow \sqrt{(x_1 + x_2)^2 - 4x_1x_2 + (y_1 + y_2)^2 - 4y_1y_2} = \sqrt{270}$$

$$0-4\left(-\frac{3\lambda^2}{2}\right)+(2)^2-4\left(1-6\lambda^2\right)=270$$

$$6\lambda^2+4-4+24\lambda^2=270$$

$$30\lambda^2=270 \Rightarrow \lambda^2=9$$

$$\lambda^2=9$$

Similarly

10. The value of D is _____

Key: 77.14

Sol: Slope of $\perp r$ bisector of $\overleftrightarrow{RS}=\frac{-1}{2}$

Midpoint of RS = $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$=\left(0,\frac{2}{2}\right)=(0,1)$

Equation $\perp r$ bisector is

$$y-1=-\frac{1}{2}(x-0)=y=\frac{-1}{2}x+1 \quad .....(4)$$

(4)Cuts curve (1) at $R'\&S'$

$$\Rightarrow 2x^2-(y-1)^2=3\lambda^2=27 \qquad \Rightarrow 2x^2-\left(-\frac{1}{2}x\right)^2=27$$

$$\Rightarrow 2x^2-\frac{x^2}{4}=27 \Rightarrow 7x^2=108 \quad ....(5)$$

$$x_1+x_2=0,\ x_1x_2=\frac{-108}{7}$$

$$\& \ 2\times4(y-1)^2-(y-1)^2=27$$

$$7(y-1)^2=27 \Rightarrow 7y^2-14y-20=0$$

$$y_1+y_2=2, y_1y_2=\frac{-20}{7}$$

$$R'S' = \sqrt{(x_1+x_2)^2 - 4x_1x_2 + (y_1+y_2)^2 - 4y_1y_2}$$

$$= \sqrt{0 - 4\left(-\frac{108}{7}\right) + 4 - 4\left(\frac{-20}{7}\right)} \qquad = \sqrt{\frac{4\times108+28+80}{7}}$$

$$D = R'S' = \sqrt{\frac{432+108}{7}} = \sqrt{\frac{540}{7}} = \sqrt{77.142} = 8.783$$

D= 8.783

SECTION 3

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get −2 marks.

11. For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let

$$E = \begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 8 & 13 & 18\end{bmatrix},\ P = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix} \text{ and } F = \begin{bmatrix}1 & 3 & 2\\ 8 & 18 & 13\\ 2 & 4 & 3\end{bmatrix}.$$

If Q is a nonsingular matrix of order 3×3, then which of the following statements is(are) TRUE?

(A) $F = PEP$ and $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $\left|EQ + PFQ^{-1}\right| = \left|EQ\right| + \left|PFQ^{-1}\right|$

(C) $\left|(EF)^3\right| > \left|EF\right|^2$.

(D) Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$.

Key: ABD

Sol:

A. $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad E = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{pmatrix}$

$F = \begin{pmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{pmatrix}$

$P^2 = I$

$PEP = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{pmatrix} = F$

$PEP = F$

$P^2EP = PF \Rightarrow EP = PF$

B. $\left|EQ + PFQ^{-1}\right| = \left|EQ + EPQ^{-1}\right|$

$= |E|\left|Q + PQ^{-1}\right|$

$= 0 \qquad (|E| = 0$

RHS = $|EQ| + \left|PFQ^{-1}\right|$

$= 0 + \left|EPQ^{-1}\right| = 0 \qquad (\because EP = PF)$

$\left|EQ + PFQ^{-1}\right| = |EQ| + \left|PFQ^{-1}\right|$

C. since $|E| = 0$

$\left|(EF)^3\right| = 0$

$|EF|^2 = 0$

$\left|(EF)^3\right| > \left|(EF)^2\right|$ is false

D. $P^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$P^{-1}EP = F$

$F + P^{-1}EP = F + F = 2F$

$$E + P^{-1}FP = E + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 4 & 6 \\ 4 & 6 & 8 \\ 16 & 26 & 36 \end{pmatrix}$$

$Tr(2F) = Tr\left(E + P^{-1}FP\right) = 44$

12. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}$$

Then which of the following statements is (are) TRUE ?

(A) f is decreasing in the interval $(-2,-1)$

(B) f is increasing in the interval $(1,2)$

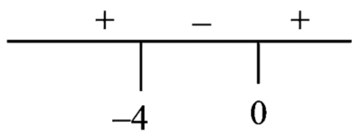
(C)f is onto

(D) Range of f is $\left[-\frac{3}{2}, 2\right]$

Key: AB

Sol: $f(x) = y = \dfrac{x^2 - 3x - 6}{x^2 + 2x + 4}$

$f'(x) = \dfrac{5x(x+4)}{\left(x^2 + 2x + 4\right)^2}$



f is increasing

$(-\infty,-4)\cup(0,\infty)$ and

decreasing $(-4,0)$

$y=\dfrac{x^2-3x-6}{x^2+2x+4}$ $\quad (y-1)x^2+(2y+3)x+4y+6=0$

$y\neq 1, \Delta\geq 0$ $\quad \Rightarrow 12y^2-4y-33\leq 0$

$y\in\left[\dfrac{-3}{2},\dfrac{11}{6}\right]-\{1\}$.

13. Let E, F and G be three events having probabilities

$P(E)=\dfrac{1}{8}, P(F)=\dfrac{1}{6}$ and $P(G)=\dfrac{1}{4}$, and let $P(E\cap F\cap G)=\dfrac{1}{10}$.

For any event H, if H^c denotes its complement, then which of the following statements is(are) TRUE?

(A) $P(E\cap F\cap G^c)\leq\dfrac{1}{40}$

(B) $P(E^C\cap F\cap G)\leq\dfrac{1}{15}$

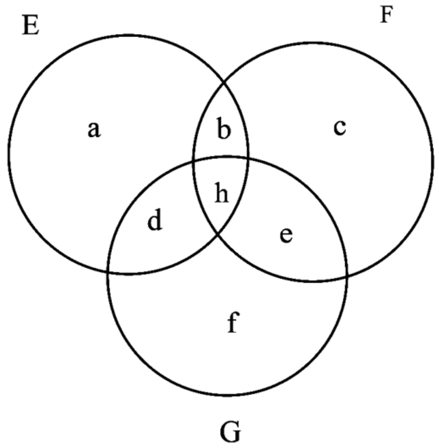
(C) $P(E\cup F\cup G)\leq\dfrac{13}{24}$

(D) $P(E^c\cap F^c\cap G^c)\leq\dfrac{5}{12}$

Key : ABC

Sol: $P(E)=\dfrac{1}{8}, P(G)=\dfrac{1}{4} P(F_1)=\dfrac{1}{6}$

$P(E\cap F_1\cap G)=\dfrac{1}{10}$



$P(E)=a+b+d+h=\frac{1}{8}$ $\quad =a+b+d=\frac{1}{8}-\frac{1}{10}=\frac{1}{40}$ …………..(1)

$P(F)=b+c+h+e=\frac{1}{6}$

$\Rightarrow$ b+c+e = $\frac{1}{6}-\frac{1}{10}=\frac{1}{15}$ …………..(2)

$P(G)=\frac{1}{4}=d+h+e+f$

$\Rightarrow d+e+f=\frac{1}{4}-\frac{1}{10}=\frac{3}{20}$ …………….(3)

From (1) $b\le\frac{1}{40}$

$P\left(E\cap F\cap G^c\right)\le\frac{1}{40}$

From (2) $e\le\frac{1}{15}$

$P\left(F\cap G\cap E^c\right)\le\frac{1}{15}$

$P\left(E\cup F\cup G\right)\le\frac{1}{8}+\frac{1}{4}+\frac{1}{6}=\frac{13}{24}$

$P\left(E\cup F\cup G\right)\le\frac{13}{24}$

$P\left(E^c\cap G^c\cap F^c\right)\ge 1-P\left(E\cup F\cup G\right)$

$\ge 1-\frac{13}{24}$ $\qquad \ge\frac{11}{24}$

14. For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let I be the 3×3 identity matrix. Let E and F be two 3×3 matrices such that $(I-EF)$ is invertible. If $G=(I-EF)^{-1}$, then which of the following statements is (are) TRUE ?

(A) $|FE|=|I-FE||FGE|$ (B) $(I-FE)(1+FGE)=I$

(C) $EFG=GEF$ (D) $(I-FE)(I-FGE)=I$

Key: ABC

Sol: (A) $(1-EF)G=I=G(I-EFG)$

$\Rightarrow G-EFG=I=G-GEF$ $\Rightarrow EFG=GEF$

(B) $G=(I-EF)^{-1}$

$\Rightarrow FGE=F(I-EF)^{-1}E$

$=\left(E^{-1}(I-EF)F^{-1}\right)^{-1}$

$\Rightarrow FGE=\left(E^{-1}F^{-1}-I\right)^{-1}$

$\Rightarrow|FGE|\cdot\left|E^{-1}F^{-1}-I\right|=1$

$\Rightarrow|FGE|\cdot\left|E^{-1}F^{-1}-I\right||FE|=|FE|$

$\Rightarrow|FGE||I-FE|=|FE|$

$\Rightarrow|FGE||I-FE|=|FE|$

(C) $(1-FE).(I+FGE)$

$=I+FGE-FE-FE.FGE=0$

$=I+FGE-FE-F(G-I)E$

$=I+FGE-FE-FGE+FE=I$

(B) $|FGE|\cdot|I-FE|=|FE|$.

(D) $(I-FE).(I-FGE)=I$.

15. For any positive integer n, let $S_n : (0,\infty) \to \mathbb{R}$ be defined by

$$S_n(x) = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1+k(k+1)x^2}{x}\right),$$

where for any $x \in \mathbb{R}$, $\cot^{-1}(x) \in (0,\pi)$ and $\tan^{-1}(x) \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then which of the following statement is (are) TRUE?

(A) $S_{10}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1+11x^2}{10x}\right)$, for all $x > 0$

(B) $\lim\limits_{n\to\infty} \cot(S_n(x)) = x$, for all $x > 0$

(C) The equation $S_3(x) = \frac{\pi}{4}$ has a root in $(0,\infty)$

(D) $\tan(S_n(x)) \le \frac{1}{2}$, for all $n \ge 1$ and $x > 0$

Key: AB

Sol: $S_n : (0,\infty) \to R$

$$S_n(x) = \sum_{k=1}^{n} \tan^{-1}\left(\frac{x}{1+k(k+1)x^2}\right)$$

$$S_n(x) = \sum_{k=1}^{n} \tan^{-1}\left(\frac{(k+1)x - kx}{1+k(kH)x^2}\right) = \sum_{k=1}^{n} \tan^{-1}(KH)x$$

$$S_n = \tan^{-1}(n+1)x - \tan^{-1}x.$$

$$S_{10}(x) = \tan^{-1}11x - \tan^{-1}x$$

$$= \tan^{-1}\left(\frac{10x}{1+11x^2}\right) = \frac{\pi}{2} - \cot^{-1}\left(\frac{10x}{1+11x^2}\right)$$

(A) $= \frac{\pi}{2} - \tan^{-1}\left(\frac{1+11x^2}{10x}\right)$

(B) $\lim\limits_{n\to\infty} \cot\left(\tan^{-1}(n+1)x - \tan^{-1}x\right)$

$$= \lim_{n\to\infty} \cot\left(\tan^{-1}\left(\frac{nx}{1+(n+1)x^2}\right)\right) = \cot\tan^{-1}\left(\frac{x}{x^2}\right)$$

$$= \cot\cot x = x$$

(C) $S_3(x) = \frac{\pi}{4} \Rightarrow \tan^{-1} 4x \tan^{-1} x = \frac{\pi}{4}$

$$\tan^{-1}\left(\frac{3x}{1+4x^2}\right) = \frac{\pi}{4} \Rightarrow \frac{3x}{1+4x^2} = 1 \Rightarrow 4x^2 - 3x + 1 = 0$$

$$9 - 4\times 4 < 0$$

(D) $\frac{nx}{1+(n+1)x^2} = \frac{1}{\frac{1}{nx} + x + \frac{x}{n}}$

$$= \frac{n}{1+n\times 1} = 1$$

16. For any complex number $w = c + id$, let $\arg(w) \in (-\pi, \pi]$, where $i = \sqrt{-1}$. Let α and β be real numbers such that for all complex numbers $z = x + iy$ satisfying $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$, the ordered pair (x, y) lies on the circle

$$x^2 + y^2 + 5x - 3y + 4 = 0$$

Then which of the following statements is (are) TRUE?

(A) $\alpha = -1$ (B) $\alpha\beta = 4$ (C) $\alpha\beta = -4$ (D) $\beta = 4$

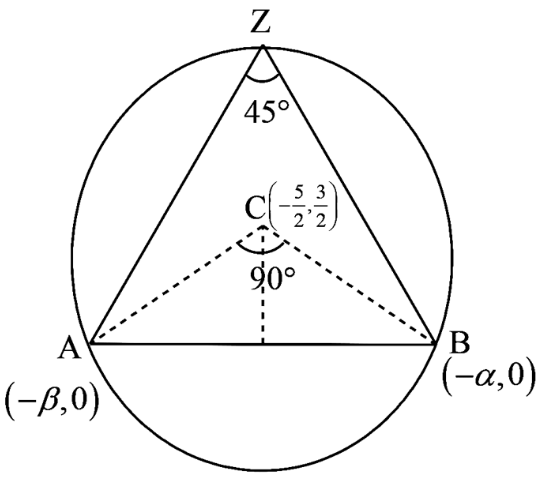
Key: BD

Sol: $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$

$y = 0$

$x^2 + 5x + 4 = 0$

$x = -4, -1$



SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

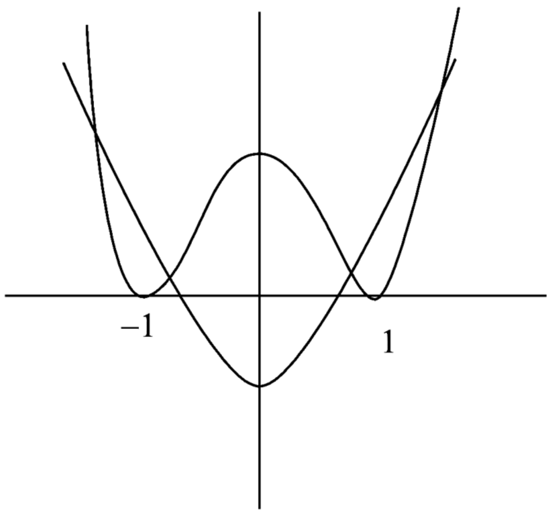
  Full Marks : +4 If ONLY the correct integer is entered;

  Zero Marks : 0 In all other cases.

17. For $x \in \mathbb{R}$, the number of real roots of the equation $3x^2 - 4\left|x^2 - 1\right| + x - 1 = 0$ is ____.

Key: 4

Sol: $3x^2 - 4\left|x^2 - 1\right| + x - 1 = 0$



$3x^2 + x - 1 = 4|x^2 - 2|$

18. In a triangle ABC, let $AB = \sqrt{23}$, BC = 3 and CA = 4. Then the value of $\frac{\cot A + \cot C}{\cot B}$ is _____

Key: 2

Sol: $AB = \sqrt{23}$, $BC = 3$, $CA = 4$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\frac{\cot A + \cot C}{\cot B} = \frac{2b^2}{a^2 + c^2 - a^2} = 2$$

19. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors in three-dimensional space, where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other and

$\vec{u} \cdot \vec{w} = 1,\ \vec{v} \cdot \vec{w} = 1,\ \vec{w} \cdot \vec{w} = 4$

If the volume of the parallelepiped, whose adjacent sides are represented by the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$, is $\sqrt{2}$, then the value of $|3\vec{u} + 5\vec{v}|$ is ______

Key: 7

Sol: $|\vec{u}| = |\vec{v}|$ $\qquad \vec{u} \cdot \vec{v} \neq 0$

$\vec{u}.\vec{w} = 1; \vec{v}.\vec{w} = 1$

$\vec{w}.\vec{w} = 4$

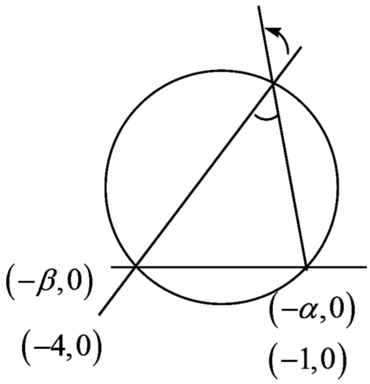
$$[\vec{u}\ \vec{v}\ \vec{w}]^2 = \begin{vmatrix} \vec{u}\cdot\vec{u} & \vec{u}.\vec{v} & \vec{u}.\vec{w} \\ \vec{v}.\vec{u} & \vec{v}.\vec{v} & \vec{v}.\vec{w} \\ \vec{v}.\vec{w} & \vec{v}.\vec{w} & \vec{w}.\vec{w} \end{vmatrix} = 2$$

$$\begin{vmatrix} 1 & \vec{u}.\vec{v} & 1 \\ \vec{u}.\vec{v} & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix} = 2$$

$$4 + \vec{u}.\vec{v} + \vec{u}.\vec{v} - 1 - 1 - 4(\vec{u}.\vec{v})^2 = 2$$

$4(\vec{u}.\vec{v}) - 2\vec{u}.\vec{v} = 0;\ \vec{u}.\vec{v} \neq 0 \Rightarrow \vec{u}.\vec{v} = \frac{1}{2}$

$|3\vec{u} + 5\vec{v}| = \sqrt{9 + 25 + 30 \times \frac{1}{2}} = \sqrt{34 + 15} = 7$



$y = 0;\ x^2 + 5x + 4 = 0$